Fixed-Time Stabilization of a Class of Strict-Feedback Nonlinear Systems via Dynamic Gain Feedback Control

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***Abstract*—This paper presents a novel fixed-time stabilization control (FSC) method for a class of strict-feedback nonlinear systems involving unmodelled system dynamics. The key feature of the proposed method is the design of two dynamic parameters. Specifically, a set of auxiliary variables is first introduced through state transformation. These variables combine the original system states and the two introduced dynamic parameters, facilitating the closed-loop system stability analyses. Then, the two dynamic parameters are delicately designed by utilizing the Lyapunov method, ensuring that all the closed-loop system states are globally fixed-time stable. Compared with existing results, the "explosion of complexity" problem of backstepping control is avoided. Moreover, the two designed dynamic parameters are dependent on system states rather than a time-varying function, thus the proposed controller is still valid beyond the given fixed-time convergence instant. The effectiveness of the proposed method is demonstrated through two practical systems.**

***Index Terms*—Dynamic gain feedback control, fixed-time stabilization, strict-feedback nonlinear system.**

I. Introduction

FIXED-TIME stabilization control (FSC), which can ensure system trajectories to converge to zero before a given time regardless of the initial conditions, has been extensively studied in the past decades [1]. This kind of control method was firstly formulated by Polyakov [2] in which the stabilization problem for uncertain linear plants is considered. It is shown that FSC can provide fast response speed together with a high control precision. Meanwhile, FSC is also able to deal with uncertain disturbances and inherent nonlinear dynamics. Because of these properties, FSC has been widely used in many mechanical and electromechanical systems [3]–[7].

The design of FSC can be divided into two categories: the time-dependent control and the state-dependent control. The time-dependent control is also called the pre-specified time control [8], [9], in which a time-varying function is used to regulate the converging rate. This function would converge to zero or infinity at the pre-specified time, causing the converging rate to be infinite. Although this kind of control can render system states to converge to zero at any time, the controller becomes invalid after the given settling time. To deal with this problem, a switching strategy has to be adopted which complicates the system control design [10], [11].

For the state-dependent control design, various technologies have been introduced in the existing literature. A hybrid control algorithm was introduced by combining a finite-time stabilizing control and a fixed-time attracting control in [2]. To avoid the chattering regimes from hybrid controls, a non-hybrid control strategy with an involution operation sign was developed for both linear and nonlinear systems in [12]. Through proposing a condition on a state-dependent function, Hua *et al.* [13] proposed a continuous FSC method for nonlinear systems. The implicit Lyapunov functions were introduced in [14] to construct fixed-time observers. Recently, Sun *et al.* [15] studied the fixed-time fuzzy tracking control problem for a class of unknown nonlinear systems. The fixed-time stabilization problem for linear systems with input delay was also studied in [16]. However, it should be noted that the control structures of the aforementioned results all possess complicated forms which are not easy to implement.

The dynamic gain control approach has been widely used to solve the stabilization problems of nonlinear systems. This approach is able to cope with system uncertainties with the desired control performance guaranteed. With the help of dynamic gain control approach, the stabilization problems were solved in [17]–[21]. It is shown in [17]–[21] that the control designed by the dynamic gains has a simple linear form which can greatly simplify the controller design. This motivates us to design FSC control method for uncertain nonlinear systems by using the dynamic gain control approach in this paper. In particular, the main contributions of this paper are summarized as below:

1) The proposed controller consists of two dynamic parameters and has a simple quasi-linear form. The two dynamic parameters are delicately designed by utilizing the Lyapunov method, ensuring that all the closed-loop system states are fixed-time stable. Compared with existing results, the "explosion of complexity" problem of backstepping control [9] is

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successfully avoided.

2) The designed controller can keep operating beyond the given fixed-time instant without any control strategy switching. This is different from the prescribed-time control methods in [9] and [11], where a time-varying function is employed to regulate the system performance. Our controller is particularly useful to cope with the case when the pre-specified time is not accurately determined.

The rest of this paper is organized as follows: Section II formulates the fixed-time stabilization problem for the strict-feedback nonlinear system, while Section III details the design of the control method and analyzes the system performance. After that, the proposed control method is verified in Section IV through two actual systems. Finally, Section V gives the conclusion remarks.

II. Preliminaries and Problem Formulation

Consider the strict-feedback nonlinear system

$$\begin{cases} \dot{x}_1 = x_2 + f_1(x_1) \\ \dot{x}_2 = x_3 + f_2(x_1, x_2) \\ \quad\vdots \\ \dot{x}_n = u + f_n(x_1, x_2, \ldots, x_n) \end{cases} \tag{1}$$

where $x = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$ is the system state, and $u \in \mathbb{R}$ is the system input. The initial time instant is set as 0, and the initial system state is denoted as $x(0)$. $f_1(\cdot)$ to $f_n(\cdot)$ are unknown nonlinear functions satisfying the following assumption [22], [23].

Assumption 1: For any $x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$, it holds that

$$|f_i(x_1, x_2, \ldots, x_i)| \le c(|x_1| + |x_2| + \cdots + |x_i|) \tag{2}$$

for $i = 1, 2, \ldots, n$, where c is a positive constant.

System (1) under Assumption 1 is a typical strict-feedback nonlinear system which has been widely studied. For example, its asymptotic stabilization problems have been solved via the dynamic gain feedback control method [21], [23], [24] or the backstepping design method [25], [26]. The fixed-time stabilizing problem was also studied in [27]–[29] through backstepping design method. This paper will develop a dynamic gain control method to achieve fixed-time stabilization whose definition is given below:

Definition 1 (Globally Fixed-Time Stable [2], [30]): Consider

$$\dot{x}(t) = g(x(t)) \tag{3}$$

where $x(t) \in \mathbb{R}^n$ is the system state, and $g(\cdot): \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function satisfying $g(0) = 0$. The initial time instant is assumed as 0 and the initial state is denoted as $x_0 \in \mathbb{R}^n$. System (3) is globally finite-time stable at the equilibrium $x = 0$ if it is Lyapunov stable and finite-time attractive, i.e., there exists a local bounded function $T(x_0): \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$ such that $x(t; x_0) = 0$ for all $t \ge T(x_0)$, where $x(t; x_0)$ is the solution of (3) with the initial state $x_0 \in \mathbb{R}^n$. The function $T(x_0)$ is called the settling time function. It is said that system (3) is globally fixed-time stable at the equilibrium $x = 0$ if it is globally finite-time stable and the settling time function $T(x_0)$ is globally bounded by some positive constant $T_{\max} > 0$, i.e., $T(x_0) \le T_{\max}, \forall x_0 \in \mathbb{R}^n$. The constant $T_{\max}$ is called the setting time.

With the above definition in mind, *the control objective of this paper* is to design the control signal u such that system (1) is globally fixed-time stable at the equilibrium $x = 0$. To this end, the following two Lemmas are given for the controller design later.

Lemma 1 ([31]): Let $0 < \tau < 1$, and $a, b \ge 0$. It holds that

$$(a+b)^\tau \le a^\tau + b^\tau. \tag{4}$$

Lemma 2 ([20]): Let a, b, γ be positive real numbers, and $\tau \in (0, 1)$. Then, the following inequality holds:

$$-\frac{a^2}{b^{1-\tau}} \le -\frac{1}{1+\tau}\gamma^{-1} a^{1+\tau} + \frac{1-\tau}{1+\tau}\gamma^{-\frac{2}{1-\tau}} b^{1+\tau}. \tag{5}$$

III. Main Results

A. Control Design

System (1) can be expressed in the matrix form

$$\dot{x} = Ax + Bu + F(x) \tag{6}$$

where

$$A = \begin{pmatrix} 0_{(n-1)\times 1} & I \\ 0 & 0_{1\times(n-1)} \end{pmatrix}, \quad B = \begin{pmatrix} 0_{(n-1)\times 1} \\ 1 \end{pmatrix} \tag{7}$$

and $F(x) = \begin{pmatrix} f_1(x_1) & f_2(x_1, x_2) & \cdots & f_n(x_1, x_2, \ldots, x_n) \end{pmatrix}^T$.

Let

$$D = \text{diag}\{1, 2, \ldots, n-1, n\}. \tag{8}$$

Then, we compute the parameters through the following algorithm.

Algorithm 1 Static Parameters for the FSC

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1 According to [32], find a vector $K = (k_1, k_2, \ldots, k_n)^T \in \mathbb{R}^n$ which meets

$$(A - BK)^T P + P(A - BK) \le -I$$
$$PD + DP \ge 0.$$

2 Denote α_1, α_2 as the minimum and maximum eigenvalues of matrix P, respectively;

3 Find a constant τ which belongs to the interval $\left(\max\left\{1 - (2n+2)\frac{\alpha_1}{\lambda_{\max}(PD+DP)}, 0\right\}, 1\right)$, where $\lambda_{\max}(\cdot)$ is the maximum eigenvalue of a matrix;

4 Find a constant β to meet

$$\beta \ge \lambda_{\max}((2n+2)P - (1-\tau)(PD+DP)).$$

It is noted that from Algorithm 1 that the following inequality holds:

$$0 < (2n+2)P - (1-\tau)(PD+DP) \le \beta I. \tag{9}$$

To make system (1) fixed-time stable, the control input u is designed as

$$u = -k_1 \frac{r_2^n}{r_1^{n(1-\tau)}} x_1 - k_2 \frac{r_2^{n-1}}{r_1^{(n-1)(1-\tau)}} x_2 - \cdots - k_n \frac{r_2}{r_1^{1-\tau}} x_n \tag{10}$$

where r_1, r_2 are the dynamic parameters to be designed. It is worth pointing out that controller (10) is in a quasi-linear form which has a simple structure and is easy to implement, compared with existing control strategies using the backstepping design method.

B. Design of The Dynamic Parameters

Before designing the dynamic parameters, a set of new variables is introduced as below

$$z_i = \frac{x_i}{r_1^{n+1-i+i\tau} r_2^i}, \quad i = 1,2,\dots,n. \tag{11}$$

Let $z = (z_1, z_2, \dots, z_n)^T \in \mathbb{R}^n$. Then, the dynamics r_1, r_2 are designed as

$$\begin{cases} \dot{r}_1 = -\frac{1}{4\beta} r_1^\tau r_2 + \frac{r_2}{4\beta r_1^{2-\tau}} \min\left\{\|z\|^2, 1\right\} \\ \dot{r}_2 = \frac{r_2^2}{2\alpha_2 r_1^{1-\tau}} \max\left\{\frac{\|z\|^{2\mu}}{r_2^{\mu+1}} - 1, 0\right\} \\ \quad 0 < r_1(0) \le 1, \ \ r_2(0) \ge \max\{8cn\|P\|, 1\} \end{cases} \tag{12}$$

where β is a constant given in (9), and $\mu \in (0, +\infty)$ is a constant to regulate the settling time T.

Now we discuss the dynamics of r_1, r_2. Because $0 < r_1(0) \le 1$ and $r_2(0) \ge \max\{8cn\|P\|, 1\}$, one has $\dot{r}_2 \ge 0$ and thus $r_2(t) \ge 1$. When $r_2 \ge \|z\|^{\frac{2\mu}{\mu+1}}$, we have $\dot{r}_2 = 0$ so that r_2 converges to a constant. When $r_2 < \|z\|^{\frac{2\mu}{\mu+1}}$, since it holds

$$\dot{r}_2 = \frac{r_2^2}{2\alpha_2 r_1^{1-\tau}} \left(\frac{\|z\|^{2\mu}}{r_2^{\mu+1}} - 1\right) > 0 \tag{13}$$

r_2 keeps increasing towards $\|z\|^{\frac{2\mu}{\mu+1}}$. Therefore, r_2 is increasing to approximate $\|z\|^{\frac{2\mu}{\mu+1}}$ when $r_2 \le \|z\|^{\frac{2\mu}{\mu+1}}$.

For the dynamic r_1, when $\|z\| \ge 1$, it holds

$$\dot{r}_1 = -\frac{r_2}{4\beta} \frac{1}{r_1^{2-\tau}} \left(r_1^2 - 1\right). \tag{14}$$

It can be seen that the equilibrium is $r_1 = 1$. Thus, we can deduce into $r_1(t) \in (0, 1]$ from $r_1(0) \in (0, 1]$. When $\|z\| \le 1$, we have

$$\dot{r}_1 = -\frac{r_2}{4\beta r_1^{2-\tau}} (r_1^2 - \|z\|^2) \tag{15}$$

which means r_1 converges towards $\|z\|$. Therefore, $r_1 \in [0, 1]$ always holds.

C. Stability Analysis

With the above controller and parameter dynamics given, we are ready to present the following theorem.

Theorem 1: Consider system (1) under Assumption 1. If the controller u is designed as (10) with the dynamic parameters r_1, r_2 given by (12), in which the vector K, matrix P, and constants α_1, α_2, β, τ are determined by Algorithm 1, then system (1) is globally fixed-time stable with a settling time T for any initial state $x(0)$. Moreover, the settling time T satisfies

$$T \le \frac{2\left(\alpha_2 + \frac{\beta}{2}\right)^{\frac{1-\tau}{2}}}{\rho(1-\tau)} + \frac{2\alpha_2}{\alpha_1} + \frac{2\alpha_2^{\mu+1}}{\mu} \tag{16}$$

where $\rho = \min\left\{\frac{1}{4(1+\tau)\alpha_2^{\frac{1+\tau}{2}}} \left(\frac{1+\tau}{2(1-\tau)}\right)^{\frac{1-\tau}{2}}, \frac{1}{2^{\frac{5-\tau}{2}} \beta^{\frac{1+\tau}{2}}}\right\}$.

Proof: Consider the variable $z = (z_1, z_2, \dots, z_n)^T$ with z_1 to z_n defined in (11). Then, we have

$$\dot{z} = \frac{r_2}{r_1^{1-\tau}} (A - BK) z - \frac{\dot{r}_1}{r_1} D_1 z - \frac{\dot{r}_2}{r_2} D_2 z + \tilde{F} \tag{17}$$

where the matrices A and B are given in (7), $D_1 = (n+1)I - (1-\tau)D$, $D_2 = D$, and $\tilde{F} = \left(\frac{f_1(\cdot)}{r_1^{n+\tau} r_2} \ \cdots \ \frac{f_n(\cdot)}{r_1^{1+n\tau} r_2^n}\right)^T$.

From (9), it can be obtained that P meets the following conditions:

$$\begin{cases} P(A - BK) + (A - BK)^T P \le -I \\ 0 < PD_1 + D_1 P \le \beta I \\ PD_2 + D_2 P \ge 0 \\ \alpha_1 I \le P \le \alpha_2 I. \end{cases} \tag{18}$$

Let $V = z^T P z$. Then its derivative along with (17) can be computed as

$$\begin{aligned} \dot{V}|_{(17)} = & \frac{r_2}{r_1^{1-\tau}} z^T \left(P(A - BK) + (A - BK)^T P\right) z \\ & - \frac{\dot{r}_1}{r_1} z^T (PD_1 + D_1 P) z \\ & - \frac{\dot{r}_2}{r_2} z^T (PD_2 + D_2 P) z + 2 z^T P \tilde{F}. \end{aligned} \tag{19}$$

It can be seen from (12) that $\dot{r}_1 \ge -\frac{1}{4\beta} r_1^\tau r_2$, $\dot{r}_2 \ge 0$, $r_1(t) \ge 0$, and $r_2(t) \ge 1$. Thus, we have

$$\begin{aligned} \dot{V}|_{(17)} \le & -\frac{r_2}{r_1^{1-\tau}} \|z\|^2 + \frac{1}{4\beta} \frac{r_2}{r_1^{1-\tau}} z^T (PD_1 + D_1 P) z \\ & + 2 z^T P \tilde{F} \\ \le & -\frac{3 r_2}{4 r_1^{1-\tau}} \|z\|^2 + 2 z^T P \tilde{F}. \end{aligned} \tag{20}$$

Following Assumption 1, the nonlinear term $\tilde{F}$ satisfies:

$$\begin{aligned} & \left|\frac{1}{r_1^{n+1-i+i\tau} r_2^i} f_i(x_1, x_2, \dots, x_i)\right| \\ & \le c \frac{|x_1|}{r_1^{n+1-i+i\tau} r_2^i} + c \frac{|x_2|}{r_1^{n+1-i+i\tau} r_2^i} + \dots + c \frac{|x_i|}{r_1^{n+1-i+i\tau} r_2^i} \\ & \le c \frac{r_1^{(i-1)(1-\tau)} |z_1|}{r_2^{i-1}} + c \frac{r_1^{(i-2)(1-\tau)} |z_2|}{r_2^{i-2}} + \dots + c |z_i| \\ & \le c|z_1| + c|z_2| + \dots + c|z_i| \\ & \le c \sqrt{n} \|z\| \end{aligned} \tag{21}$$

which indicates $\|\tilde{F}\| \le cn\|z\|$. Combining (20) and (22) yields

$$\begin{aligned} \dot{V}|_{(17)} \le & -\frac{3 r_2}{4 r_1^{1-\tau}} \|z\|^2 + 2cn\|P\| \|z\|^2 \\ \le & -\frac{3 r_2}{4 r_1^{1-\tau}} \|z\|^2 + \frac{2cn\|P\|}{r_1^{1-\tau}} \|z\|^2 \\ \le & -\frac{r_2}{2 r_1^{1-\tau}} \|z\|^2 \end{aligned} \tag{22}$$

where $r_2(t) \ge r_2(0) \ge 8cn\|P\|$ is employed.

The following proof is divided into four parts. Part I shows the fixed-time attractivity of z, while the fixed-time convergence of z is guaranteed in Part II. The fixed-time convergence of x is derived from the dynamic of z in Part III, and the upper bound of the settling time T is estimated in Part IV.

Part I: Fixed-Time Attractivity of System (17)

In this part, we show that for any initial condition $z(0)\in\mathbb{R}^n$, $z(t)$ will converge to a neighbourhood $\Omega=\{z \mid \|z\|^2\le 1\}$ before a given time T_0.

Consider $\omega=\frac{V}{r_2}$. If $r_2<\|z\|^{\frac{2\mu}{\mu+1}}$, r_2 in (12) satisfies

$$\dot r_2=\frac{r_2^2}{2\alpha_2 r_1^{1-\tau}}\left(\frac{\|z\|^{2\mu}}{r_2^{\mu+1}}-1\right) \tag{23}$$

and

$$\begin{aligned}\dot\omega&=\frac{\dot V}{r_2}-\frac{\dot r_2}{r_2^2}V\\ &\le -\frac{1}{2r_1^{1-\tau}}\|z\|^2-\frac{1}{2\alpha_2 r_1^{1-\tau}}\left(\frac{\|z\|^{2\mu}}{r_2^{\mu+1}}-1\right)V\\ &\le -\frac{1}{2\alpha_2 r_1^{1-\tau}}\frac{\|z\|^{2\mu}}{r_2^{\mu}}\frac{V}{r_2}\end{aligned} \tag{24}$$

where $\|z\|^2\ge\frac{V}{\alpha_2}$ is utilized.

If $r_2\ge\|z\|^{\frac{2\mu}{\mu+1}}$, then r_2 in (12) satisfies $\dot r_2=0$. Thus r_2 becomes a constant, and the derivative of ω is

$$\dot\omega\le-\frac{1}{2\alpha_2 r_1^{1-\tau}}V\le-\frac{1}{2\alpha_2 r_1^{1-\tau}}\frac{\|z\|^{2\mu}}{r_2^{\mu}}\frac{V}{r_2}. \tag{25}$$

To conclude, whether $r_2<\|z\|^{\frac{2\mu}{\mu+1}}$ or $r_2\ge\|z\|^{\frac{2\mu}{\mu+1}}$, it can be deduced from (24) and (25) that

$$\dot\omega\le-\frac{1}{2\alpha_2 r_1^{1-\tau}}\frac{\|z\|^{2\mu}}{r_2^{\mu}}\frac{V}{r_2}. \tag{26}$$

Since $r_1\in[0,1]$, one obtain $\dot\omega\le-\frac{1}{2\alpha_2^{\mu+1}}\omega^{\mu+1}$, and

$$\begin{aligned}&\omega(t)\le\left(\frac{1}{\frac{1}{\omega^{\mu}(0)}+\frac{\mu}{2\alpha_2^{\mu+1}}t}\right)^{\frac{1}{\mu}}, && \omega(0)>0\\ &\omega(t)=0, && \omega(0)=0.\end{aligned}$$

When $t\ge\frac{2\alpha_2^{\mu+1}}{\mu}$, it holds $\omega(t)\le 1$. Back to (22), we obtain

$$\dot V|_{(17)}\le-\frac{1}{2}\|z\|^2V\le-\frac{1}{2\alpha_2}V^2. \tag{27}$$

If $V(\frac{2\alpha_2^{\mu+1}}{\mu})\ne 0$, one can get

$$V(t)\le\frac{1}{\frac{1}{V(\frac{2\alpha_2^{\mu+1}}{\mu})}+\frac{1}{2\alpha_2}(t-\frac{2\alpha_2^{\mu+1}}{\mu})}$$

and

$$\|z(t)\|^2\le\frac{1}{\frac{\alpha_1}{V(\frac{2\alpha_2^{\mu+1}}{\mu})}+\frac{\alpha_1}{2\alpha_2}(t-\frac{2\alpha_2^{\mu+1}}{\mu})}.$$

Therefore, when $t\ge T_0=\frac{2\alpha_2}{\alpha_1}+\frac{2\alpha_2^{\mu+1}}{\mu}$, $z(t)$ will be in the neighborhood Ω, i.e., $\|z(t)\|^2\le 1$, $\forall t\ge T_0$. Meanwhile, by noticing $r_2\ge 1$, after the time instant T_0, it holds that $\dot r_2(t)=0$, which means $r_2(t)$ will converge to a constant when $t\ge T_0$.

Part II: Fixed-Time Stability of System (17)

From Part I, we have $\|z(t)\|\le 1$ for $t\ge T_0$. This part will show that under this condition, a time instant T can be found which guarantees $z(t)=0$ for any $t\ge T$. Since $\frac{\|z\|^{2\mu}}{r_2^{\mu+1}}-1<0$ holds, the parameter r_2 becomes constant. As a result, the parameter r_1 satisfies

$$\dot r_1=-\frac{1}{4\beta}r_1^{\tau}r_2+\frac{r_2}{4\beta r_1^{2-\tau}}\|z\|^2. \tag{28}$$

Letting $\varpi=V+\frac{\beta}{2}r_1^2$, its derivative can be computed as

$$\begin{aligned}\dot\varpi&\le-\frac{r_2}{2r_1^{1-\tau}}\|z\|^2-\frac{1}{4}r_1^{1+\tau}r_2+\frac{r_2}{4r_1^{1-\tau}}\|z\|^2\\ &\le-\frac{1}{4r_1^{1-\tau}}\|z\|^2-\frac{1}{4}r_1^{1+\tau}\end{aligned} \tag{29}$$

where $r_2\ge 1$ is employed.

Employing Lemma 2 and choosing $\gamma=\left(\frac{1+\tau}{2(1-\tau)}\right)^{-\frac{1-\tau}{2}}$, we can get

$$-\frac{1}{4r_1^{1-\tau}}\|z\|^2\le-\frac{\gamma^{-1}}{4(1+\tau)}\|z\|^{1+\tau}+\frac{(1-\tau)\gamma^{-\frac{2}{1-\tau}}}{4(1+\tau)}r_1^{1+\tau}.$$

Then, from (29) we can obtain

$$\begin{aligned}\dot\varpi&\le-\frac{1}{4(1+\tau)\alpha_2^{\frac{1+\tau}{2}}}\gamma^{-1}V^{\frac{1+\tau}{2}}-\frac{1}{8}r_1^{1+\tau}\\ &\le-\rho V^{\frac{1+\tau}{2}}-\rho\frac{\beta_2^{\frac{1+\tau}{2}}}{2^{\frac{1+\tau}{2}}}r_1^{1+\tau}\end{aligned} \tag{30}$$

where $\rho=\min\left\{\frac{1}{4(1+\tau)\alpha_2^{\frac{1+\tau}{2}}}\gamma^{-1},\frac{1}{2^{\frac{5-\tau}{2}}\beta^{\frac{1+\tau}{2}}}\right\}$.

Employing Lemma 1 to (30) gives

$$\dot\varpi\le-\rho(V+\frac{\beta}{2}r_1^2)^{\frac{1+\tau}{2}}\le-\rho\varpi^{\frac{1+\tau}{2}}. \tag{31}$$

Then, it holds

$$\varpi(t)\le\left(\varpi^{\frac{1-\tau}{2}}(T_0)-\frac{1-\tau}{2}\rho(t-T_0)\right)^{\frac{2}{1-\tau}} \tag{32}$$

for $t\in[T_0,T]$. Therefore, $\varpi(t)=0$ when $t>T=T_0+\varpi^{\frac{1-\tau}{2}}(T_0)\frac{2}{(1-\tau)\rho}$. This can be further deduced into

$$\lim_{t\to T}\|z(t)\|=0,\quad \lim_{t\to T}r_1(t)=0. \tag{33}$$

Thus, at the time instant T, $\|z(t)\|$ and $r_1(t)$ converge to zero.

Part III: Fixed-Time Stability of System (1)

From the definition of z in (11), it holds

$$\begin{cases}x_1=r_1^{n+\tau}r_2z_1\\ x_2=r_1^{n-1+2\tau}r_2^2z_2\\ \quad\vdots\\ x_n=r_1^{1+n\tau}r_2^nz_n.\end{cases} \tag{34}$$

Meanwhile, from (22), we have

$$\|z(t)\|^2 \le \frac{1}{\alpha_1}V(t) \le \frac{1}{\alpha_1}V(0). \tag{35}$$

The parameter r_2 can be estimated as

$$\begin{aligned} r_2(t) &\le \max\{\max_{s\in[0,T]} \|z(s)\|^{\frac{2\mu}{\mu+1}}, r_2(0)\} \\ &\le \max\left\{\left(\frac{1}{\alpha_1}V(0)\right)^{\frac{\mu}{\mu+1}}, r_2(0)\right\}. \end{aligned} \tag{36}$$

Therefore, following (33) and (37) yields:

$$\lim_{t\to T}\|x(t)\| = 0.$$

Moreover, under the new variable $z = (z_1, z_2, \ldots, z_n)^T$, the control signal u can be expressed as

$$u(t) = -r_1^{(n+1)\tau} r_2^{n+1} Kz. \tag{37}$$

As r_1, r_2 and z are all bounded, thus u is bounded all the time. Moreover, from (34) we have $\lim_{t\to T}\|z(t)\| = 0$.

It can also be seen that

$$\begin{aligned} \|x(t)\|^2 &\le r_2^{2n}\|z(t)\|^2 \\ &\le \frac{1}{\alpha_1}\max\left\{\left(\frac{1}{\alpha_1}V(0)\right)^{\frac{2n\mu}{\mu+1}}, r_2^{2n}(0)\right\}V(0). \end{aligned} \tag{38}$$

As $V(0)$ is determined by the initial state $\|x(0)\|$ and $r_2(0)$ is chosen by the designer, we can conclude that the upper bounded of $\|x(t)\|$ is governed by the initial system state $x(0)$.

Part IV: Estimation of the Settling Time

As discussed above, the FSC introduced in this paper contains three stages. The first stage is to ensure $\omega(t) \le 1$, which happens when $t \ge \frac{2\alpha_2^{\mu+1}}{\mu}$. Then, under the condition $\omega(t) \le 1$, we can get $\|z(t)\| \le 1$ when $t \ge T_0$ with $T_0 = \frac{2\alpha_2}{\alpha_1} + \frac{2\alpha_2^{\mu+1}}{\mu}$. At last, from $\|z(t)\| \le 1$, it holds that $\varpi(t) = 0$ when $t \ge T$ with $T = T_0 + \varpi^{\frac{1-\tau}{2}}(T_0)\frac{2}{(1-\tau)\rho}$.

Since $\varpi(T_0) \le \alpha_2 + \frac{\beta}{2}$, we have

$$T \le \frac{2\left(\alpha_2 + \frac{\beta}{2}\right)^{\frac{1-\tau}{2}}}{\rho(1-\tau)} + \frac{2\alpha_2}{\alpha_1} + \frac{2\alpha_2^{\mu+1}}{\mu} \tag{39}$$

which is not dependent on the system initial conditions. Therefore, the designed controller (10) and (12) can render system (1) globally fixed-time stable. ■

Remark 1: Our control strategy does not suffer from the singularity problem. It can be seen from (13) that $\|z\|$, i.e., $\|x\|$, converges to zero before r_1 does. Therefore, even though r_1 tends to zero, the singularity problem for the control signal u does not happen. Indeed, it is proved in Theorem 1 that the control signal is always bounded and converges to zero in the end.

Remark 2: Our proposed control method is essentially different from the prescribed-time control methods in [9] and [11] in the following aspects: 1) Our considered system model is more general than the counterparts in [9] and [11]. Specifically, the system model in [9] contains nonlinear dynamics that exist only in the last differential equation, while the system model in [11] has no nonlinear dynamics; 2) The control action of our method does not have to terminate at the prespecified time instant T while the methods in [9] and [11] must. This is because [9] and [11] adopt a monotonically increasing function to achieve prescribed-time control. This function is only valid in the time interval $[t_0, t_0+T)$, thus the control method becomes invalid when $t \ge t_0 + T$. By contrast, our method does not need the above function and thus it keeps valid all the time. This property is particularly useful to cope with the case when the time instant T is not accurately determined.

IV. Simulation Results

In this paper, we present two simulation examples to verify the effectiveness of our proposed method.

Example 1: Consider the one-link manipulator system given in [33]

$$\begin{cases} D\ddot{q} + B\dot{q} + N\sin(q) = \tau_r \\ M\dot{\tau}_r + H\tau_r = u - K_m\dot{q} \end{cases} \tag{40}$$

where q, $\dot{q}$, $\ddot{q}$ are the link position, velocity and acceleration, respectively. τ_r is the torque produced by the electrical subsystem. u is the control input representing the electromechanical torque. D is the mechanical inertia, B is the coefficient of viscous friction at the joint, N is a positive constant related to the mass of the load and the coefficient of gravity, M is the armature inductance, H is the armature resistance, and K_m is the back electromotive force coefficient.

Denoting $x_1 = MDq$, $x_2 = MD\dot{q}$ and $x_3 = M\tau_r$, (40) can be transformed into

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 + f_2(x_1, x_2) \\ \dot{x}_3 = u + f_3(x_2, x_3) \end{cases} \tag{41}$$

where $f_2(x_1,x_2) = -\frac{B}{D}x_2 - MN\sin(\frac{x_1}{MD})$, $f_3(x_2,x_3) = -\frac{K_m}{MD}x_2 - \frac{H}{M}x_3$.

For simulation, the system parameters are set as $D = 5\ \mathrm{kg\cdot m^2}$, $N = 0.2$, $M = 0.5$ H, $K_m = 0.05\ \mathrm{N\cdot m/A}$, $B = 0.2\ \mathrm{N\cdot m\cdot s/rad}$, and $H = 0.02\ \Omega$. It can be verified that Assumption 1 is satisfied with $c = 0.04$. Then, the control parameters are set as $k_1 = 2$, $k_2 = 6$, $k_3 = 3$, $\tau = 1/2$, $\beta = 20.7$, $\alpha_1 = 0.2$, $\alpha_2 = 5$, $\mu = 3$. In particular, according to Theorem 1, the controller for this one-link manipulator system is designed as

$$u = -2r_2^3 r_1^{-\frac{3}{2}} x_1 - 6r_2^2 r_1^{-1} x_2 - 3r_2 r_1^{-\frac{1}{2}} x_3 \tag{42}$$

where r_1, r_2 are the dynamic parameters designed as

$$\begin{cases} \dot{r}_1 = -\dfrac{1}{4\times 20.7} r_1^{\frac{1}{2}} r_2 + \dfrac{r_2}{4\times 20.7 r_1^{\frac{3}{2}}} \min\left\{\|z\|^2, 1\right\} \\ \dot{r}_2 = \dfrac{r_2^2}{2\times 5 r_1^{\frac{1}{2}}} \max\left\{\dfrac{\|z\|^6}{r_2^4} - 1, 0\right\} \\ r_1(0) = 0.9, \quad r_2(0) = 2.62 \end{cases} \tag{43}$$

with $z = \left(x_1/(r_1^{\frac{7}{2}} r_2),\ x_2/(r_1^3 r_2^2),\ x_3/(r_1^{\frac{5}{2}} r_2^3)\right)^T$.

The simulation results are presented in Fig. 1. It is observed that the system states x_1, x_2 and x_3 converge to zero before the instant $T = 4$ s. Meanwhile, the dynamic parameter r_1 remains in the interval $[0,1]$ and converges to zero, while r_2 increases to about 3.4 and then remains the same. The control signal u is also bounded and converges to zero. Moreover, it can be seen that the closed-loop system can still operate beyond the time interval $[0,4]$, which cannot be achieved through the pre-specified finite-time control method in [9] and [11].

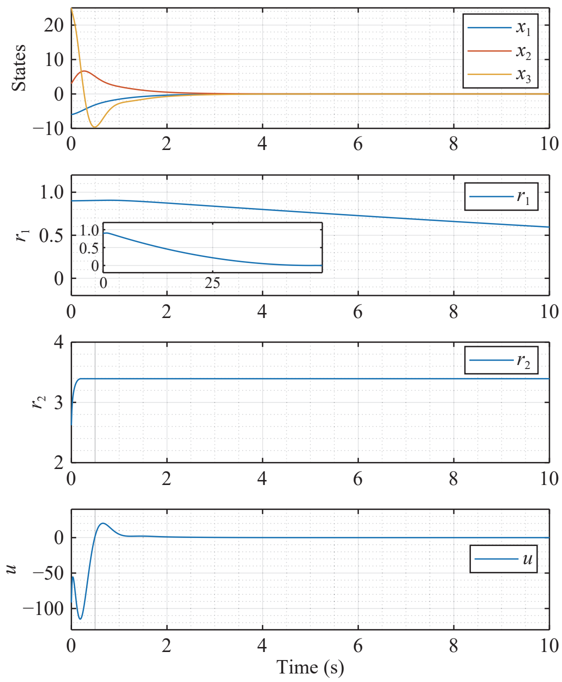

Fig. 1. Simulation results for closed-loop system (41)–(43) with initial state $x(0) = (-6, 3, 25)^T$.

Example 2: In this example, we consider a more complex robotic manipulator coupled to a DC motor to further confirm the effectiveness of our proposed method. The system model [34] is given as below

$$\begin{cases} J_1\ddot{q}_1 + F_1\dot{q}_1 + K(q_1 - \frac{q_2}{N}) + mgd\cos q_1 = 0 \\ J_2\ddot{q}_2 + F_2\dot{q}_2 - \frac{K}{N}(q_1 - \frac{q_2}{N}) = K_t i \\ L\dot{i} + Ri + K_b\dot{q}_2 = u \end{cases} \tag{44}$$

where the physical meanings of the relevant parameters are displayed in Table I. To facilitate the controller design, the following coordinate transformation is introduced:

$$\begin{cases} x_1 = q_1 \\ x_2 = \dot{q}_1 \\ x_3 = \frac{K}{J_1 N}q_2 - \frac{mgd}{J_1}\cos(q_1) \\ x_4 = \frac{K}{J_1 N}\dot{q}_2 \\ x_5 = \frac{KK_t}{J_1 J_2 N}i - \frac{mgdK}{J_1 J_2 N^2}\cos(q_1) \\ u = \frac{mgdR}{K_t N}\cos(q_1) + u^*. \end{cases} \tag{45}$$

TABLE I
EXPLANATION OF PARAMETERS

Parameters	Physical meanings
q_1	Angular position of the link
q_2	Motor shaft
m	Link mass
g	Acceleration of gravity
i	Armature current
L	Armature inductance
R	Armature resistance
u	Armature voltage
d	Position of the link's center of gravity
N	Gear ratio
K_b	Back EMF constant
K	Spring constant
K_t	Torque constant
J_1, J_2	Inertias constants
F_1, F_2	Viscous friction constants

Consequently, we have

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 - \frac{F_1}{J_1}x_2 - \frac{K}{J_1}x_1 \\ \dot{x}_3 = x_4 + \frac{mgd}{J_1}x_2\sin(x_1) \\ \dot{x}_4 = x_5 - \frac{F_2}{J_2}x_4 + \frac{K^2}{J_1 J_2 N^2}x_1 - \frac{K}{J_2 N^2}x_3 \\ \dot{x}_5 = -\frac{R}{L}x_5 - \frac{K_b K_t}{J_2 L}x_4 + \frac{mgdKx_2\sin(x_1)}{J_1 J_2 N^2} + \frac{K_t K u^*}{J_1 J_2 N L}. \end{cases} \tag{46}$$

For the sake of simulation, the system parameters are chosen as $F_1 = F_2 = 1$ N·m·s/rad, $J_1 = J_2 = 1$ kg·m^2, $K = 1$ kgf/mm, $N = 1$, $R = 1\ \Omega$, $L = 1$ H, $K_b = 1$ V/rad/s, $K_t = 1$ N·m, $g = 10$ m/s^2, $m = 0.5$ kg, and $d = 0.2$. The system's initial condition is set as $x(0) = [x_1(0), x_2(0), x_3(0), x_4(0), x_5(0)] = [1,1,1,1,1,1]$, and $[r_1(0), r_2(0)] = [1,1]$.

The control objective is to stabilize system (46). It can be verified that (46) satisfies Assumption 1, thus our control method can be applied. To show the effectiveness of our proposed method, we also compare our method with the one in [34]. The control parameters of our method are set as $k_1 = 6$, $k_2 = k_5 = 5$, $k_3 = 8.5$, $k_4 = 10$, $\beta = 20.7$, $\tau = 0.5$, $\alpha_1 = 0.2$, $\alpha_2 = 5$, and $\mu = 3$, while the parameters of [34] remain the same as those given in the simulation examples of [34].

The simulation results are presented in Fig. 2. As can be observed, the system's angular with our method converges to zero with a faster speed than the one in [34]. Moreover, the amplitude of our control signal is also significantly smaller. This clearly demonstrates the advantage of our proposed method compared with the one in [34].

V. CONCLUSION

This paper has proposed a dynamic gain control method to

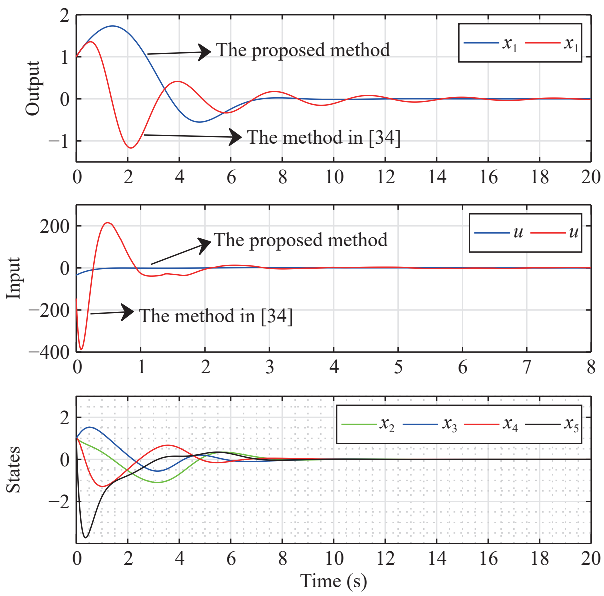

Fig. 2. Simulation results.

solve the fixed-time control problem for a class of strict-feedback nonlinear systems. The considered systems can describe a large class of practical systems with unmodelled dynamics satisfying a linear-growth condition. To handle the unmodelled system dynamics, two auxiliary parameters are delicately designed. Our design method yields a quasi-linear controller which ensures that the closed-loop system is fixed-time stable. Compared with existing results, our proposed method avoids the "explosion of complexity" problem caused by conventional backstepping control. Moreover, the designed controller can keep operating beyond the given fixed-time instant without any control strategy switching, which is superior to the conventional prescribed-time control. Two simulation examples have verified the effectiveness of our proposed method.

References

[1] Y. Liu, H. Li, Z. Zuo, X. Li, and R. Lu, "An overview of finite/fixed-time control and its application in engineering systems," *IEEE/CAA J. Autom. Sinica*, vol. 9, no. 12, pp. 2106–2120, 2022.

[2] A. Polyakov, "Nonlinear feedback design for fixed-time stabilization of linear control systems," *IEEE Trans. Automat. Control*, vol. 57, no. 8, pp. 2106–2110, Aug. 2012.

[3] F. L. Chernousko, I. M. Ananievski, and S. A. Reshmin, "Control of nonlinear dynamical systems: Methods and applications," Springer Science & Business Media, 2008.

[4] M. Defoort, A. Polyakov, G. Demesure, M. Djemai, and K. Veluvolu, "Leader-follower fixed-time consensus for multi-agent systems with unknown non-linear inherent dynamics," *IET Control Theory Appli.*, vol. 9, no. 14, pp. 2165–2170, Sep. 2015.

[5] H. Hong, W. Yu, G. Wen, and X. Yu, "Distributed robust fixed-time consensus for nonlinear and disturbed multiagent systems," *IEEE Trans. Syst. Man, Cybern. Syst.*, vol. 47, no. 7, pp. 1464–1473, Jul. 2016.

[6] B. Ning and Q.-L. Han, "Prescribed finite-time consensus tracking for multiagent systems with nonholonomic chained-form dynamics," *IEEE Trans. Automat. Control*, vol. 64, no. 4, pp. 1686–1693, Apr. 2019.

[7] L. Zhang, Y. Xia, G. Shen, and B. Cui, "Fixed-time attitude tracking control for spacecraft based on a fixed-time extended state observer," *Sci. China Inform. Sci.*, vol. 64, no. 11, pp. 212201:1–212201:17, Oct. 2021.

[8] B. Ning, Q.-L. Han, and Z. Zuo, "Practical fixed-time consensus for integrator-type multi-agent systems: A time base generator approach," *Automatica*, vol. 105, pp. 406–414, Jul. 2019.

[9] Y. Song, Y. Wang, J. Holloway, and M. Krstic, "Time-varying feedback for regulation of normal-form nonlinear systems in prescribed finite time," *Automatica*, vol. 83, pp. 243–251, Sep. 2017.

[10] L. Chang, Q.-L. Han, X. Ge, C. Zhang, and X. Zhang, "On designing distributed prescribed finite-time observers for strict-feedback nonlinear systems," *IEEE Trans. Cyber.*, vol. 51, no. 9, pp. 4695–4706, Sep. 2021.

[11] Y. Wang, Y. Song, D. J. Hill, and M. Krstic, "Prescribed-time consensus and containment control of networked multiagent systems," *IEEE Trans. Cyber.*, vol. 49, no. 4, pp. 1138–1147, Apr. 2019.

[12] J. Li, Y. Yang, C. Hua, and X. Guan, "Fixed-time backstepping control design for high-order strict-feedback non-linear systems via terminal sliding mode," *IET Control Theory Appli.*, vol. 11, no. 8, pp. 1184–1193, Sep. 2016.

[13] C. Hua, Y. Li, and X. Guan, "Finite/fixed-time stabilization for nonlinear interconnected systems with dead-zone input," *IEEE Trans. Automat. Control*, vol. 62, no. 5, pp. 2554–2560, May 2017.

[14] F. Lopez-Ramirez, A. Polyakov, D. Efimov, and W. Perruquetti, "Finite-time and fixed-time observer design: Implicit Lyapunov function approach," *Automatica*, vol. 87, pp. 52–60, Feb. 2018.

[15] Y. Sun, F. Wang, Z. Liu, Y. Zhang, and C. L. Chen, "Fixed-time fuzzy control for a class of nonlinear systems," *IEEE Trans. Cyber.*, vol. 52, no. 5, pp. 3880–3887, May 2022.

[16] B. Zhou, W. Michiels, and J. Chen, "Fixed-time stabilization of linear delay systems by smooth periodic delayed feedback," *IEEE Trans. Automat. Control*, vol. 67, no. 2, pp. 557–573, Feb. 2022.

[17] L. Praly, "Asymptotic stabilization via output feedback for lower triangular systems with output dependent incremental rate," *IEEE Trans. Automat. Control*, vol. 48, no. 6, pp. 1103–1108, Jun. 2003.

[18] L. Praly and Z. Jiang, "Linear output feedback with dynamic high gain for nonlinear systems," *Syst. Control Letters*, vol. 53, no. 2, pp. 107–116, Oct. 2004.

[19] Q. Liu and Z. Liang, "Finite-time consensus of time-varying nonlinear multi-agent systems," *Int. J. Syst. Sci.*, vol. 47, no. 11, pp. 2642–2651, Aug. 2016.

[20] X. Zhang, G. Feng, and Y. Sun, "Finite-time stabilization by state feedback control for a class of time-varying nonlinear systems," *Automatica*, vol. 48, no. 3, pp. 499–504, Mar. 2012.

[21] D. Fan, X. Zhang, Y. Chang, and X. Lu, "Global practical tracking via disturbance rejection control for uncertain nonlinear systems with quantized input and output," *Sci. China Inform. Sci.*, vol. 65, pp. 119201:1–119201:3, Feb. 2021.

[22] M. S. Koo, H. L. Choi, and J. T. Lim, "Global regulation of a class of uncertain nonlinear systems by switching adaptive controller," *IEEE Trans. Autom. Control*, vol. 55, no. 12, pp. 2822–2827, Dec. 2010.

[23] H. L. Choi and J. T. Lim, "Stabilization of a class of nonlinear systems by adaptive output feedback," *Automatica*, vol. 41, no. 6, pp. 1091–1097, Jun. 2005.

[24] C. Qian and W. Lin, "Output feedback control of a class of nonlinear systems: A nonseparation principle paradigm," *IEEE Trans. Automat. Control*, vol. 47, no. 10, pp. 1710–1715, Oct. 2002.

[25] W. Wang, J. Long, J. Zhou, J. Huang, and C. Y. Wen, "Adaptive backstepping based consensus tracking of uncertain nonlinear systems with event-triggered communication," *Automatica*, vol. 133, pp. 109841:1–109841:9, Aug. 2021.

[26] W. Chang, Y. Li, and S. Tong, "Adaptive fuzzy backstepping tracking control for flexible robotic manipulator," *IEEE/CAA J. Autom. Sinica*, vol. 8, no. 12, pp. 1923–1930, Dec. 2021.

[27] Z. Gao and G. Guo, "Command filtered finite/fixed-time heading tracking control of surface vehicles," *IEEE/CAA J. Autom. Sinica*, vol. 8, no. 10, pp. 1667–1776, Oct. 2021.

[28] J. Liu, Y. Wu, M. Sun, and C. Sun, "Fixed-time cooperative tracking for delayed disturbed multi-agent systems under dynamic event-triggered control," *IEEE/CAA J. Autom. Sinica*, vol. 9, no. 5, pp. 930–933, May

2022.

[29] F. Wang and G. Lai, "Fixed-time control design for nonlinear uncertain systems via adaptive method," *Syst. Control Letters*, vol. 140, pp. 104704:1–104740:7, Jun. 2020.

[30] S. Bhat and S. D. Bernstein, "Finite-time stability of continuous autonomous systems," *SIAM J. Control Optim.*, vol. 38, no. 3, pp. 751–766, Oct. 2000.

[31] C. Qian and W. Lin, "Non-Lipschitz continuous stabilizers for nonlinear systems with uncontrollable unstable linearization," *Syst. Control Letters*, vol. 42, no. 3, pp. 185–200, Mar. 2001.

[32] P. Krishnamurthy and F. Khorrami, "On uniform solvability of parameter-dependent Lyapunov inequalities and applications to various problems," *SIAM J. Control Optim.*, vol. 45, no. 4, pp. 1147–1164, Sep. 2006.

[33] W. Li, X. Yao, and M. Krstic, "Adaptive-gain observer-based stabilization of stochastic strict-feedback systems with sensor uncertainty," *Automatica*, vol. 120, pp. 109112:1–109112:12, Jul. 2020.

[34] Z.-L. Zhao and Z.-P. Jiang, "Semi-global finite-time output-feedback stabilization with an application to robotics," *IEEE Trans. Ind. Electron.*, vol. 66, no. 4, pp. 3148–3156, Apr. 2019.

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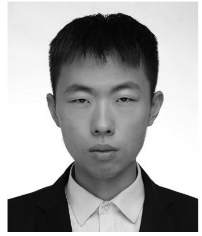

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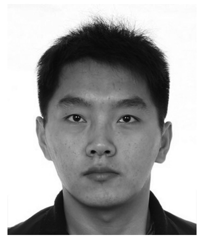

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