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Global output feedback stabilisation for nonlinear systems via a switching control gain approach

Le Chang[a], Cheng Fu[b] and Huanjun Zhang[c]

[a]College of Electric Power Engineering, Shanghai University of Electric Power, Shanghai, People's Republic of China; [b]School of Automation, Qingdao University, Qingdao, People's Republic of China; [c]School of Mathematics and Statistics, Shandong Normal University, Jinan, People's Republic of China

ABSTRACT

This paper presents a novel approach to design the output feedback stabilising controller for a class of nonlinear systems. The considered system is in the strict-feedback form, and the nonlinear terms are allowed to depend on both the unmeasured states and the measured output. Meanwhile, the growth rate of the nonlinear term can be a general continuous function of the output, which is not required to be polynomial. We introduce a switching control gain approach and design the controller through considering the output feedback stabilisation based on a linear dynamic. It avoids a continuous update in a control gain. The effectiveness of this method is illustrated through a presented example in the end.

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Strict-feedback nonlinear systems; high gain stabilising control; switching control gains; event triggered; output stabilisation

1. Introduction and problem statement

In this paper, we consider the output feedback stabilisation problem for the nonlinear system described by

$$\begin{aligned} \dot{x}_i &= x_{i+1} + f_i(x_1, x_2, \ldots, x_i), \quad i = 1, 2, \ldots, n-1, \\ \dot{x}_n &= u + f_n(x_1, x_2, x_3, \ldots, x_n), \\ y &= x_1, \end{aligned} \tag{1}$$

where $x = (x_1, x_2, \ldots, x_n)^\top \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}$ is the system input and $y \in \mathbb{R}$ is the system output. The initial instant is t_0, and the initial state is denoted as $x(t_0)$. The functions f_1–f_n are continuous and satisfying the following assumption.

Assumption 1.1: For any $x = (x_1, x_2, \ldots, x_n)^\top \in \mathbb{R}^n$, it holds

$$\left|f_i(x_1, x_2, \ldots, x_i)\right| \leq \phi(x_1)\left(|x_1| + |x_2| + \cdots + |x_i|\right) \tag{2}$$

for $i = 1, 2, \ldots, n$, where $\phi(\cdot) : \mathbb{R} \to \mathbb{R}^+$ is a continuous function.

The global output feedback stabilisation problem of nonlinear system (1) is an important topic in control theory which has been investigated by Khalil (2002), Mazenc and Bowong (2004) and Mazenc et al. (2018). The high gain/dynamic gain feedback control is the main technology in the study of this topic. For instance, Choi and Lim (2005), Lei and Lin (2006) and Qian and Lin (2002, 2004) designed the high gain feedback control for the constant growth rate case, i.e. $\phi(x_1)$ is a constant in (2). For a more complex case, Benabdallah et al. (2014), Liang et al. (2022), Praly and Jiang (2004), Shang et al. (2011) and Wang et al. (2021) considered the output-dependent polynomial growth rate, i.e. $\phi(x_1) = (1 + x_1^p)$ with $p \geq 1$ being a positive integer. Krishnamurthy and Khorrami (2024) employed the dynamic high gain feedback control to stabilise the feedforward nonlinear systems. It was shown that the linear stabilising controller could still be provided by designing a dynamic gain, and this gain was turned to be a bounded constant. But when $\phi(x_1)$ is a general smooth continuous function, the control design problem becomes a challenging task since the bound of the dynamic gain cannot be guaranteed.

There are some existing results focusing on the control design problem when $\phi(x_1)$ is a smooth continuous function.

By introducing a reduced-order observer, Krishnamurthy and Khorrami (2007) solved this problem by ensuring x_1 to be bounded, and then the dynamic gain was designed to regulate the system state into zero. When the strict-feedback nonlinear systems include uncertainties and unknown time-varying delays, Li et al. (2023) proposed the regulating controller based on a reduced-order observer with a dynamic gain. For the large-scale systems consisting multi strict-feedback nonlinear subsystems, Zhang, Liu et al. (2013) and Zhang and Lin (2015) designed the controller based on the reduced-order observer and dynamic gain feedback control approach. The reduced-order observer is to estimate the states $x_2, x_3, \ldots, x_n$ without the output $y = x_1$, and then the output y is an extra state which is turned to be bounded. In this case, the bound of x_1 is guaranteed, and the dynamic gain would converge to a finite constant. But the reduced-order observer is generally in nonlinear form, which is not easy to be understood.

This paper will design a controller with switching gains to stabilise the nonlinear system (1), and the framework is depicted in Figure 1. The switching control gains have advantages over studying the nonlinear systems (Chang & Fu, 2022; Hespanha

CONTACT Le Chang lchang@shiep.edu.cn

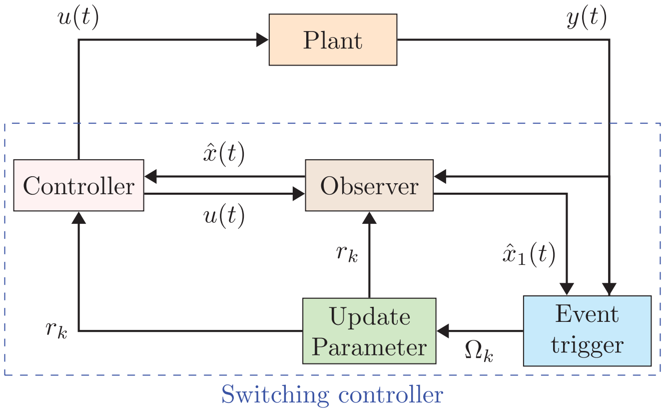

Figure 1. The diagram of control loop in this paper.

et al., 2003; Koo et al., 2010; Ye, 2005). In these results, a logic-based switching is often used to select a suitable controller among candidates. In Hespanha et al. (2003), it is shown that a logic-based switching can be utilised to overcome the limitations of conventional adaptive control for the system with uncertainty. The method in Ye (2005) was designing the control gains to be switched between candidates under a switch logic, and it was applicable to nonlinear systems whose nonlinearities only depend on the output. By considering a switching control gain, Koo et al. (2010) tuned the dynamic gain depending on the nonlinearity structure, and stabilising or regulating the system with unknown nonlinearity structure. In Chang and Fu (2022), the switching control gain was designed to achieve the stabilising controller for nonlinear systems with unknown saturation. Although the switching control gains have such advantages, it is still needed to answer whether the nonlinear systems (1) can be stabilised by a switching control.

We will design the event-triggered logic to update the constant gains such that the gains can be learned or updated to a desired constant. In this case, the resulting controller has a simple form, and it is easy to be understood by the designer. Thus the contributions of this paper can be summarised as below.

- *The nonlinear system in our paper is more general*, when compared with the results in Benabdallah et al. (2014), Liang et al. (2022), Praly and Jiang (2004), Shang et al. (2011) and Wang et al. (2021). These results were considering the output-dependent polynomial growth rate $1 + |x_1|^p$, while we consider the general growth rate $\varphi(x_1)$. The output-depended polynomial growth rate can be seen as a special case of our system.
- *We give a novel switching-parameter method to design the stabilising controller*. The existing results by Li et al. (2023), Krishnamurthy and Khorrami (2007), Zhang, Liu et al. (2013) and Zhang and Lin (2015) were focusing on the dynamic gain parameter with a reduced-order observer, and it separated the output $y = x_1$ with other states to ensure the bound of the output. Different from their approaches, our designed control is based on the classical high-gain observer with a switched parameter. We will prove that the controller based on the high-gain observer can stabilise system (1) via designing the high gain parameter as a switching mechanism.

The rest of this paper is organised as follows: Section 2 presents the designed controller, and the system performance is analysed in Section 3. Section 4 gives an example to illustrate the effectiveness of our proposed method, while Section 5 presents the conclusion remarks. A reference list ends this paper.

Notation: Denote by $\mathbb{R}$ the field of real numbers and $\mathbb{R}^{m\times n}$ the set of $m \times n$ real matrices. The notation $\|\cdot\|$ is the Euclidean norm for vectors or the induced Euclidean norm of matrices. I stands for the identity matrix of appropriate dimensions. For a symmetric matrix P, we use $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ to denote the largest and smallest eigenvalues of P. The argument of functions will be omitted or simplified whenever no confusion can arise from the context. For example, we denote $x(t)$ by x.

2. Control design

The control with a parameter r is given as below:

$$u = -k_1 r^n \hat{x}_1 - k_2 r^{n-1}\hat{x}_2 - \cdots - k_n r\hat{x}_n,$$
$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 - r l_1(\hat{x}_1 - y) \\ \dot{\hat{x}}_2 = \hat{x}_3 - r^2 l_2(\hat{x}_1 - y) \\ \quad\vdots \\ \dot{\hat{x}}_n = u - r^n l_n(\hat{x}_1 - y) \end{cases} \tag{3}$$

where $\hat{x} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)^\top \in \mathbb{R}^n$ is the observer state. The coefficients $\{l_1, l_2, \ldots, l_n\}$ and $\{k_1, k_2, \ldots, k_n\}$ are constants to make the polynomials $p^n + l_1 p^{n-1} + \cdots + l_n$, $p^n + k_n p^{n-1} + \cdots + k_1$ be Hurwitz.

We design the parameter r through a switching mechanism. It contains an event-triggered instant $\{t_k\}_{k\ge 0}$ and an updated dynamic.

2.1 Event-triggered instants

The switching instants $\{t_k\}_{k\ge 0}$ are given as the triggered condition

$$t_{k+1} = \inf\left\{t > t_k \,\middle|\, |y(t)| \ge \lambda_{1,k} \left(|\hat{x}_1(t_k)| + |\hat{x}_1(t_k) - y(t_k)|\right) + c\right\}, \tag{4}$$

where $\lambda_{1,k} \ge 1$ is a dynamic satisfying $\lambda_{1,k+1} > \lambda_{1,k} \ge 1$, and c is an arbitrary positive constant. Denote $\Omega_k = [-\omega_k, \omega_k]$ where ω_k is a dynamic given as

$$\omega_k = \lambda_{1,k}\left(|\hat{x}_1(t_k)| + |\hat{x}_1(t_k) - y(t_k)|\right) + c.$$

Because $|y(t_k)| \le (|\hat{x}_1(t_k)| + |\hat{x}_1(t_k) - y(t_k)|)$ and the triggered condition, we get $y(t) \in \Omega_k$ for any instant $t \in [t_k, t_{k+1})$.

2.2 Updated dynamic

$$r(t) = \max\left\{\lambda_2 \varphi_k, \lambda_{1,k} r(t_{k-1})\right\}, \quad t \in [t_k, t_{k+1}), \tag{5}$$

where λ_2 is a positive constant, and

$$\varphi_k = \max\left\{\max_{\delta\in\Omega_k} \phi(\delta), 1\right\}.$$

To make the update dynamic well-defined, it is required to denote $r(t_{-1}) = 1$. Under our design, the parameter r satisfies

- it is non-decreasing, i.e. $r(t_1) \geq r(t_2)$ if $t_1 \geq t_2$;
- it holds $\{t_k\}_{k\geq 0}$ such that $r(t) = r(t_k)$, $t \in [t_k, t_{k+1})$;
- it is depended on $\phi(x_1)$.

Remark 2.1: The controller (3) is based on linear dynamics $\hat{x}$ and has a simple form. To solve the stabilising problem of system (1), it was designed based on the reduced-order observer with the dynamic parameter by Li et al. (2023), Zhang, Liu et al. (2013) and Zhang and Lin (2015). It was expressed as

$$\begin{cases} \dot{z}_i = z_{i+1} + r^i a_{i+1} y - (i-1)\dot{r} r^{i-2} a_i y - r^{i-1} a_i (z_2 + r a_2 y), \\ \dot{z}_n = u - (n-1)\dot{r} r^{n-2} a_n y - r^{n-1} a_n (z_2 + r a_2 y), \end{cases}$$

where a_2 to a_n are constants and r is the dynamic parameter. Then it was proved that the stabilisation problem would be solved by designing the controller as

$$u(t) = -r^n \left(k_2 \frac{\hat{z}_2 + r a_2 y + M(y)}{r} + k_3 \frac{\hat{z}_3 + r^2 a_3 y}{r^2} + \cdots + k_n \frac{\hat{z}_n + r^{n-1} a_n y}{r^{n-1}} \right)$$

with k_2 to k_n being constants, $M(y)$ be a continuous dynamic respect to the output y. Our paper is considering the high-gain observer, and we solve the stabilisation problem through designing a switching mechanism on the high-gain parameter r. It is noted that there are many extended methods based on the high-gain observer. More details can be found in Huang and Duan (2024) and Khalil (2017) and the related references therein. It gives us opportunities to using our introduced method to solve the problems for complex nonlinear systems based on the high-gain observer.

3. Main results

We consider $e_i, i = 1, 2, \ldots, n$, as the error state between the observer (3) and the system (1), i.e. $e_i = \hat{x}_i - x_i$. Then, it holds

$$\begin{cases} \dot{e}_1 = e_2 - r l_1 e_1 - f_1(x_1), \\ \dot{e}_2 = e_3 - r^2 l_2 e_1 - f_2(x_1, x_2), \\ \quad \vdots \\ \dot{e}_n = -r^n l_n e_1 - f_n(x_1, x_2, \ldots, x_n). \end{cases}$$

Let $e = (e_1, e_2, \ldots, e_n)^\top$, and we get the matrix form as

$$\dot{e} = \left(A - r^{n+1} H^{-1} LC\right) e - F(x), \tag{6}$$

where $L = (l_1, l_2, \ldots, l_n)^\top$, $H = \mathrm{diag}\{r^n, r^{n-1}, \ldots, r\}$, $C = (1, 0, 0, \ldots, 0)$ and

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix},$$

$$F(x) = \begin{pmatrix} f_1(x_1) \\ f_2(x_1, x_2) \\ \vdots \\ f_{n-1}(x_1, x_2, \ldots, x_{n-1}) \\ f_n(x_1, x_2, \ldots, x_{n-1}, x_n) \end{pmatrix}.$$

We also rewrite the observer dynamic (3) as

$$\begin{aligned} u &= -KH\hat{x}, \\ \dot{\hat{x}} &= A\hat{x} + Bu - r^{n+1} H^{-1} LCe, \end{aligned} \tag{7}$$

where $B = (0, 0, \ldots, 0, 1)^\top$, and $K = (k_1, k_2, \ldots, k_n)$.

Combining (6) and (7), we get the closed-loop system as

$$\begin{cases} \dot{e} = \left(A - r^{n+1} H^{-1} LC\right) e - F(x), \\ \dot{\hat{x}} = (A - BKH)\hat{x} - r^{n+1} H^{-1} LCe. \end{cases} \tag{8}$$

It is noted that system (8) is a switching system since the parameter r in matrix H is switching. In the sequel, we will analyse the system performance under the switching framework.

3.1 The performance during the continuous part

During the period $[t_k, t_{k+1})$, we consider the state transformation

$$\xi_1 = \hat{x}_1, \quad \xi_2 = \frac{1}{r}\hat{x}_2, \quad \ldots, \quad \xi_n = \frac{1}{r^{n-1}}\hat{x}_n \tag{9}$$

and

$$\varepsilon_1 = e_1, \quad \varepsilon_2 = \frac{1}{r} e_2, \quad \ldots, \quad \varepsilon_n = \frac{1}{r^{n-1}} e_n. \tag{10}$$

Because of r is a constant during the period $[t_k, t_{k+1})$, we can transform the closed-loop system (8) as

$$\begin{cases} \dot{\varepsilon} = r(A - LC)\varepsilon - \tilde{F}, \\ \dot{\xi} = r(A - BK)\xi - rLC\varepsilon, \end{cases} \tag{11}$$

where $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)^\top$ and $\xi = (\xi_1, \xi_2, \ldots, \xi_n)^\top$ and

$$\tilde{F} = \begin{pmatrix} f_1(x_1) \\ \frac{1}{r} f_2(x_1, x_2) \\ \vdots \\ \frac{1}{r^{n-2}} f_{n-1}(x_1, x_2, \ldots, x_{n-1}) \\ \frac{1}{r^{n-1}} f_n(x_1, x_2, \ldots, x_{n-1}, x_n) \end{pmatrix}$$

Since l_1 to l_n and k_1 to k_n are coefficients of a Hurwitz polynomial, we can conclude that matrices $A-LC$ and $A-BK$ are Hurwitz. Then we achieve two positive definite matrices P, Q such that

$$(A - BK)^\top P + P(A - BK) \leq -I,$$
$$(A - LC)^\top Q + Q(A - LC) \leq -I.$$

Let

$$V = \xi^\top P\xi + \sigma\varepsilon^\top Q\varepsilon,$$

where $\sigma = 4\|PLC\|^2 + \frac{5}{4}$, and its derivative is computed as

$$\begin{aligned}\dot{V}|_{(11)} =& r\xi^\top \left((A-BK)^\top P + P(A-BK)\right)\xi \\ &+ \sigma r\varepsilon^\top \left((A-LC)^T Q + Q(A-LC)\right)\varepsilon \\ &- 2r\xi^\top PLC\varepsilon - 2\sigma\varepsilon^\top Q\tilde{F} \\ \le& -r\|\xi\|^2 - r(4\|PLC\|^2 + \frac{5}{4})\|\varepsilon\|^2 \\ &+ 2r\|\xi\|\,\|PLC\|\,\|\varepsilon\| + 2\sigma\|\varepsilon\|\,\|Q\|\,\|\tilde{F}\| \\ \le& -\frac{3r}{4}\|\xi\|^2 - \frac{5r}{4}\|\varepsilon\|^2 + 2\sigma\|Q\|\,\|\varepsilon\|\,\|\tilde{F}\|.\end{aligned} \tag{12}$$

To continue, we consider the estimation of $\tilde{F}$. From Assumption 1.1, we get

$$\begin{aligned}&\left|\frac{1}{r^{i-1}}f_i(x_1, x_2, \ldots, x_i)\right| \\ &\le \phi(x_1)\left(|x_1| + \frac{|x_2|}{r} + \cdots + \frac{|x_i|}{r^{i-1}}\right) \\ &\le \phi(x_1)\left(|\hat{x}_1 - e_1| + \frac{|\hat{x}_2 - e_2|}{r} + \cdots + \frac{|\hat{x}_i - e_i|}{r^{i-1}}\right) \\ &\le \phi(x_1)\left((|\xi_1| + |\varepsilon_1|) + (|\xi_2| + |\varepsilon_2|) + \cdots + (|\xi_i| + |\varepsilon_i|)\right) \\ &\le \sqrt{n}\phi(x_1)\left(\|\xi\| + \|\varepsilon\|\right),\end{aligned}$$

for any $i = 1, 2, \ldots, n$. Then we achieve

$$\|\tilde{F}\| \le n\phi(x_1)\left(\|\xi\| + \|\varepsilon\|\right).$$

Substituting the above estimation into (12) to get

$$\begin{aligned}\dot{V}|_{(11)} &\le -\frac{3r}{4}\|\xi\|^2 - \frac{5r}{4}\|\varepsilon\|^2 + 2n\sigma\|Q\|\phi(x_1)\|\varepsilon\|\left(\|\xi\| + \|\varepsilon\|\right) \\ &\le -\frac{3r}{4}\|\xi\|^2 - \frac{5r}{4}\|\varepsilon\|^2 + n\sigma\|Q\|\phi(x_1)\left(\|\xi\|^2 + 3\|\varepsilon\|^2\right) \\ &\le -\frac{r}{2}\|\xi\|^2 - \frac{r}{2}\|\varepsilon\|^2 - \left(\frac{1}{4}r - n\sigma\|Q\|\phi(x_1)\right)\left(\|\xi\|^2 + 3\|\varepsilon\|^2\right).\end{aligned} \tag{13}$$

Noted that when $t \in [t_k, t_{k+1})$, it holds $y(t) \in \Omega_k$. That is, $\phi(x_1) \le \varphi_k$. Through choosing

$$\lambda_2 \ge 4n\sigma\|Q\|,$$

we get $r(t) \ge \lambda_2\varphi_k \ge 4n\sigma\|Q\|\phi(x_1)$. Then, back to (13), we achieve

$$\dot{V}|_{(11)} \le -\frac{r}{2}\|\xi\|^2 - \frac{r}{2}\|\varepsilon\|^2. \tag{14}$$

Therefore, during each continuous period $[t_k, t_{k+1})$, there exists a positive definite function V such that $\dot{V} < 0$, if $V \ne 0$.

Next, we determine the sequence $\lambda_{1,k}$ such that the switching is finite.

3.2 Finite switching

From the above analysis, we get

$$V(t_{k+1}^-) \le e^{-\frac{1}{2}\frac{1}{\max\{\lambda_{\max}(P), \sigma\lambda_{\max}(Q)\}}(t_{k+1}-t_k)}V(t_k),$$

where $V(t_{k+1}^-)$ stands for $lim_{h\to 0, h<0}V(t_{k+1}+h)$. By choosing

$$\lambda_k = \sqrt{\frac{2n\max\left\{\lambda_{\max}(P), \left(4\|PLC\|^2 + \frac{5}{4}\right)\lambda_{\max}(Q)\right\}}{\min\left\{\lambda_{\min}(P), \left(4\|PLC\|^2 + \frac{5}{4}\right)\lambda_{\min}(Q)\right\}}} \, e^{-\frac{1}{4}\frac{t_{k+1}-t_k}{\max\{\lambda_{\max}(P), \sigma\lambda_{\max}(Q)\}}} \tag{15}$$

we further deduce into

$$\begin{aligned}&\left(\sum_{i=1}^{n}|\xi_i(t_{k+1}^-)| + |\varepsilon_i(t_{k+1}^-)|\right)^2 \\ &\le 2n\left(\|\xi(t_{k+1}^-)\|^2 + \|\varepsilon\left(t_{k+1}^-\right)\|^2\right) \\ &\le \frac{2n}{\min\{\lambda_{\min}(P), \sigma\lambda_{\min}(Q)\}}V\left(t_{k+1}^-\right) \\ &\le \lambda_k^2\left(\|\xi(t_k)\|^2 + \|\varepsilon(t_k)\|^2\right) \\ &\le \lambda_k^2\left(\sum_{i=1}^{n}|\xi_i(t_k)| + |\varepsilon_i(t_k)|\right)^2\end{aligned} \tag{16}$$

On the other hand, from the design of $r(t_k)$ in (5), we get

$$r(t_{k+1}) \ge \lambda_{1,k}r(t_k).$$

Then, at each switching instant t_{k+1}, it holds

$$\begin{aligned}\left|\xi_i(t_{k+1})\right| &= \frac{r^{i-1}(t_k)}{r^{i-1}(t_{k+1})}\frac{1}{r^{i-1}(t_{k+1}^-)}\left|\hat{x}_i(t_{k+1})\right| \\ &\le \left(\frac{1}{\lambda_{1,k}}\right)^{i-1}\left|\xi_i(t_{k+1}^-)\right| \le \frac{1}{\lambda_{1,k}}\left|\xi_i(t_{k+1}^-)\right|, \\ &\quad i = 2, 3, \ldots, n,\end{aligned}$$

and

$$\begin{aligned}\left|\varepsilon_i(t_{k+1})\right| &= *\frac{r^{i-1}(t_k)}{r^{i-1}(t_{k+1})}\frac{1}{r^{i-1}(t_{k+1}^-)}\left|\varepsilon_i(t_{k+1})\right| \\ &\le \left(\frac{1}{\lambda_{1,k}}\right)^{i-1}\left|\varepsilon_i(t_{k+1}^-)\right| \le \frac{1}{\lambda_{1,k}}\left|\varepsilon_i(t_{k+1}^-)\right|, \\ &\quad i = 2, 3, \ldots, n.\end{aligned}$$

Back to (16), we get

$$\begin{aligned}&|y(t_{k+1})| + \sum_{i=2}^{n}\lambda_{1,k}\left(|\xi_i(t_{k+1})| + |\varepsilon_i(t_{k+1})|\right) \\ &\le \left(|\hat{x}_1(t_{k+1})| + |e_1(t_{k+1})|\right) \\ &+ \sum_{i=2}^{n}\lambda_{1,k}\left(|\xi_i(t_{k+1})| + |\varepsilon_i(t_{k+1})|\right)\end{aligned}$$

$$\leq \left(|\hat{x}_1(t_{k+1}^-)| + |e_1(t_{k+1}^-)|\right) + \sum_{i=2}^{n} \left(|\xi_i(t_{k+1}^-)| + |\varepsilon_i(t_{k+1}^-)|\right)$$
$$\leq \lambda_k \left(\sum_{i=1}^{n} |\xi_i(t_k)| + |\varepsilon_i(t_k)| \right). \tag{17}$$

Meanwhile, since

$$|y(t_{k+1})| \geq \lambda_{1,k} \left(|\hat{x}_1(t_k)| + |\hat{x}_1(t_k) - y(t_k)|\right) + c$$
$$\geq \lambda_{1,k} \left(|\xi_1(t_k)| + |\varepsilon(t_k)|\right) + c,$$

we get

$$\lambda_{1,k} \left(|\xi_1(t_k)| + |\varepsilon(t_k)|\right) + c + \lambda_{1,k} \sum_{i=2}^{n} \left(|\xi_i(t_{k+1})| + |\varepsilon_i(t_{k+1})|\right)$$
$$\leq \lambda_k \sum_{i=1}^{n} \left(|\xi_i(t_k)| + |\varepsilon_i(t_k)|\right).$$

Since $\lambda_{1,k}$ is an increasing sequence, we can find an index k' such that

$$\lambda_{1,k} \geq \lambda_k, \quad k \geq k'$$

we achieve

$$\frac{c}{\lambda_k} + \sum_{i=2}^{n} \left(|\xi_i(t_{k+1})| + |\varepsilon_i(t_{k+1})|\right) \leq \sum_{i=2}^{n} \left(|\xi_i(t_k)| + |\varepsilon_i(t_k)|\right). \tag{18}$$

This means, after the index k', for each switching, the value of $(|\xi_i(t_k)| + |\varepsilon_i(t_k)|)$ is decreasing at least $\frac{c}{\lambda_k}$. Then the state in the kth switching satisfies

$$\sum_{i=2}^{n} \left(|\xi_i(t_k)| + |\varepsilon_i(t_k)|\right)$$
$$\leq \sum_{i=2}^{n} \left(|\xi_i(t_{k-1})| + |\varepsilon_i(t_{k-1})|\right) - \frac{c}{\lambda_k}$$
$$\ldots$$
$$\leq \sum_{i=2}^{n} \left(|\xi_i(t_{k'})| + |\varepsilon_i(t_{k'})|\right) - (k - k')\frac{c}{\lambda_k}.$$

Since $(|\xi_i(t_k)| + |\varepsilon_i(t_k)|) \geq 0$, we conclude that the switching time satisfies

$$k \leq \frac{1}{c}\lambda_k \sum_{i=2}^{n} \left(|\xi_i(t_{k'})| + |\varepsilon_i(t_{k'})|\right) + k'$$
$$\leq \frac{1}{c}\sqrt{\frac{2n \max\{\lambda_{\max}(P), \sigma\lambda_{\max}(Q)\}}{\min\{\lambda_{\min}(P), \sigma\lambda_{\min}(Q)\}}} \sum_{i=2}^{n} \left(|\xi_i(t_{k'})| + |\varepsilon_i(t_{k'})|\right) + k'.$$

Thus we get the switching is finite. We will analyse the convergence performance next.

3.3 Convergence analysis

Denote t_d as the last switching instant, and d being the index. Then, from the event-triggered condition, we get

$$|y(t)| < \lambda_{1,k} \left(|\hat{x}_1(t_d)| + |\hat{x}_1(t_d) - y(t_d)|\right) + c, \quad t \in [t_d, +\infty).$$

Then it holds $y(t) \in \Omega_d$ and $\phi(x_1) \leq \varphi_d$. We can further conclude (14). That is

$$\dot{V}|_{(11)} \leq -\frac{r(t_d)}{2}\|\xi\|^2 - \frac{r(t_d)}{2}\|\varepsilon\|^2, \quad t \in [t_d, +\infty).$$

Since

$$V \leq \lambda_{\max}(P)\|\xi\|^2 + \sigma\lambda_{\max}(Q)\|\varepsilon\|^2,$$

we get

$$\dot{V}|_{(11)} \leq -\frac{r(t_d)}{2} \max\{\lambda_{\max}(P), \sigma\lambda_{\max}(Q)\} V,$$

which guarantees the convergence of V.

From the definition of V, we can get the convergence of ξ and ε. The parameter r is a constant during $[t_d, +\infty)$. Thus, under the transformation (9), (10), we deduce into the convergence of $\hat{x}$ and e. Because of $e = \hat{x} - x$, we conclude the convergence of x.

Therefore, our main result is summarised as below:

Theorem 3.1: *Under Assumption 1.1, the nonlinear system (1) can be stabilised through the control (3) with switching parameter $r(t)$ in (5), event-triggered condition (4).*

Remark 3.1: This result can be extended to the uncertain nonlinear systems. When the nonlinear term f_i in system (1) satisfies $f_i(x_1, x_2, \ldots, x_i)| \leq \theta\phi(x_1)(|x_1| + \cdots + |x_i|), i = 1, 2, \ldots, n$ with θ being an unknown constant, we can design the controller as

$$u = -k_1 p^n r^n \hat{x}_1 - k_2 p^{n-1} r^{n-1} \hat{x}_2 - \cdots - k_n p r \hat{x}_n,$$
$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 - r p l_1(\hat{x}_1 - y), \\ \dot{\hat{x}}_2 = \hat{x}_3 - r^2 p^2 l_2(\hat{x}_1 - y), \\ \quad\vdots \\ \dot{\hat{x}}_n = u - r^n p^n l_n(\hat{x}_1 - y), \end{cases}$$

where $p = t + 1$ is a time-varying variable, r is a switching parameter given by (4) and (5). Then, combining the time-varying method in Qian and Lin (2004) and the method in this paper, we can easily prove that the sequence $\{\lambda_{1,k}\}_{k\geq 0}$ and the constant λ_2 can be chosen such that

$$\lim_{t\to+\infty} x_i = 0, \quad \lim_{t\to+\infty} \hat{x}_i = 0, \quad i = 1, 2, \ldots, n.$$

Remark 3.2: Noted that our result gives a sufficient condition for the stabilisation of the closed-loop system (1) and (3). In many scenarios, we can ensure the convergence when the condition $\lambda_{1,k} < \lambda_k$ is not satisfied. Meanwhile, it can be deduced from (18) that

$$c\sqrt{\frac{\min\{\lambda_{\min}(P), \sigma\lambda_{\min}(Q)\}}{2n \max\{\lambda_{\max}(P), \sigma\lambda_{\max}(Q)\}}}$$

$$\le \sum_{i=2}^{n} (|\xi_i(t_k) + |\epsilon_i(t_k)|)$$
$$\le \sum_{i=2}^{n} \left(|\hat{x}_i(t_k) + |\hat{x}_i(t_k) - x_i(t_k)|\right).$$

This means when c is larger than a constant depended on the initial condition, the parameter would not update. Therefore, through appropriately choosing the parameter c can $\lambda_{1,k}$, the system performance can be improved.

Remark 3.3: The event-triggered mechanism (4) we designed is different from the event-triggered mechanism for the communication network by Tang et al. (2024) and Yang et al. (2024). Through designing the event to update parameter finite times, we can achieve the stabilising controller for any initial condition. Our event occurs finite times and thus the Zeno phenomenon is avoided.

4. An example

In this section, we present an example to illustrate the method we proposed.

Example 4.1: Consider the nonlinear system

$$\begin{aligned} \dot{x}_1 &= x_2 + 2e^{x_1}x_1, \\ \dot{x}_2 &= u - e^{x_1}x_2, \\ y &= x_1, \end{aligned} \tag{19}$$

where $x = (x_1, x_2)^T \in \mathbb{R}^2$ is the system state, $u \in \mathbb{R}$ is the system input and $y \in \mathbb{R}$ is the system output. The initial instant is assumed as 0.

It is verified that $2e^{x_1}x_1$, $e^{x_1}x_2$ are nonlinear terms satisfying Assumption 1.1 with $\phi(x_1) = 2e^{x_1}$. Thus, following our proposed method, we can design the controller as

$$\begin{aligned} u &= -r^2\hat{x}_1 - 2r\hat{x}_2, \\ \dot{\hat{x}}_1 &= \hat{x}_2 - 2r(\hat{x}_1 - y) \\ \dot{\hat{x}}_2 &= u - r^2(\hat{x}_1 - y), \end{aligned} \tag{20}$$

where $\hat{x} = (\hat{x}_1, \hat{x}_2)^T$ is the observer state with initial value $(0, 0)^T$. The parameter r is switching. The update dynamic is given as

$$r(t) = \max\left\{38e^{y_k} + 1, \quad \lambda_{1,k}r(t_{k-1})\right\}, \qquad t \in [t_k, t_{k+1}) \tag{21}$$

with $\lambda_{1,k} = 1 + 0.01k$, the instants $\{t_k\}_{k\ge 0}$ being determined by

$$t_{k+1} = \inf\left\{t > t_k \,\middle|\, |y(t)| \ge \lambda_{1,k} \left(|\hat{x}_1(t_k)| + |\hat{x}_1(t_k) - y(t_k)|\right) + 0.1\right\}. \tag{22}$$

By choosing the initial state $x_1(0) = -0.01, x_2(0) = 20, \hat{x}_1(0) = 0$, $\hat{x}_2(0) = 0$, the simulation results are shown in Figure 2 and Figure 3. It can be seen in Figure 2 that the states x_1, $\hat{x}_1$ converge to zero, and in Figure 3 that the states x_2, $\hat{x}_2$ converge to zero. In Figure 4, the control input u is depicted. The parameter r is switching three times, from 42.4 to 52.2. This illustrates the effectiveness of our method.

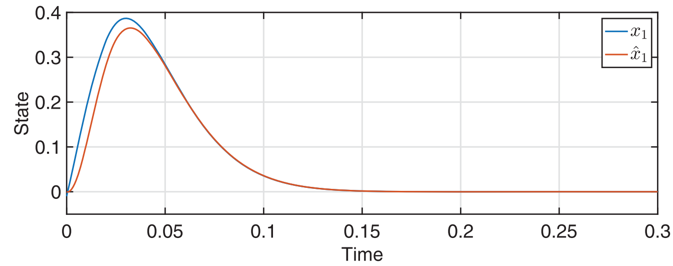

Figure 2. Trajectory of $x_1, \hat{x}_1$ in closed-loop system (19)–(22).

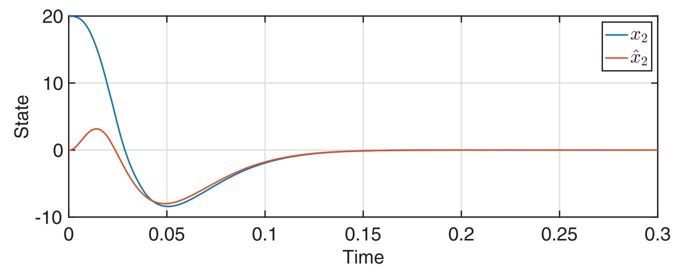

Figure 3. Trajectory of $x_2, \hat{x}_2$ in closed-loop system (19)–(22).

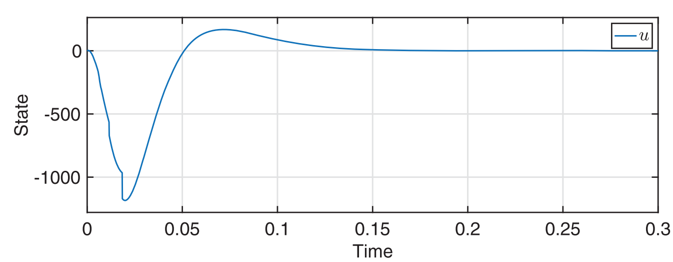

Figure 4. Trajectory of input u in closed-loop system (19)–(22).

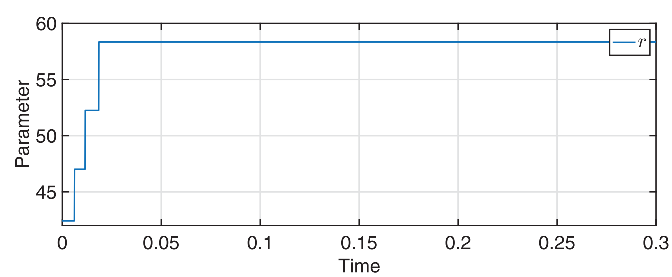

Figure 5. Trajectory of switching parameter r in closed-loop system (19)–(22).

It is noted that the controller based on the reduced-order observer is designed as

$$\begin{aligned} u &= -kr^2\left(\frac{z_2 + M(y)y}{r} + y\right) \\ \dot{z}_2 &= u - l\dot{r}y - lr(z_2 + lry) \\ \dot{r} &= \max\left\{r\omega_1(y) + \omega_2(y) - \frac{r^2}{4}, 0\right\}, \quad r(0) = 1, \end{aligned}$$

where k, l are regulated positive constants, and $M(y)$, $\omega_1(y)$, $\omega_2(y)$ are designed dynamics. Thus, compared with our controller, it has a complex form, even for a 2-order nonlinear system.

5. Conclusion

This paper designed a stabilising controller for a class of strict-feedback nonlinear systems. The nonlinear terms in the system were depended on the unmeasured states and output. We proposed a novel method to design the controller which was based on the Luenberger observer with a switching parameter. An updated law was designed for the parameter, and an event-triggered mechanism to determine the switching instants. The designed controller had a simple form, and it was easy to be understood. By considering a numerical example, the effectiveness of this method has been verified. The future work is to improve the system performance through optimise the switching parameter, and a method is needed to ensure the stability as well as updating the switching gain gradually.

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