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Global Output Regulation for Uncertain Feedforward Nonlinear Systems With Unknown Nonlinear Growth Rate

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ABSTRACT

This paper proposes a method for designing an output regulation controller for a class of feedforward nonlinear systems, where the nonlinear terms exhibit an unknown input-dependent rate. The completely unknown nature of this rate introduces significant challenges to control design, rendering existing adaptive and time-varying methods ineffective. To address these challenges, we first introduce a bound on the input and transform the nonlinear condition into an equivalent case with an unknown constant rate. Subsequently, we develop both time-varying and adaptive methods to design the controller. Through rigorous mathematical analysis, we prove that the regulation objective can be achieved under the proposed control framework. Illustrative examples are provided to demonstrate the effectiveness of the designed control strategy.

1 | Introduction

Research on feedforward nonlinear systems has been a central focus in control theory, with numerous investigations over the years. These systems are commonly encountered in practical scenarios, such as the cart-pendulum system [1] and the induction heater circuit system [2]. They often possess an upper triangular structure, making them generally non-feedback linearizable, which poses significant challenges in designing controllers.

Designing an output feedback stabilizing controller for nonlinear systems is a particularly difficult problem. It has been shown that the output feedback problems can only be solved for nonlinear systems when the nonlinear terms satisfy specific conditions [3, 4]. The Lipschitz condition is commonly considered in nonlinear systems, which ensures the existence and uniqueness of the solution. By considering the equilibrium point as zero, the Lipschitz condition can be interpreted as a linear growth condition.

Early works focused on the output feedback stabilization for feedforward nonlinear systems with the linear growth condition [5, 6], while other factors, such as delays [7, 8], measurement sensitivity [9], stochastic dynamics [7], and sampling output [9–11], were also considered.

For more complex cases, the nonlinear term was allowed to have an input-dependent incremental rate [12], where the Lipschitz coefficient becomes a function of the input. The control parameter can be designed to account for this input-dependent function. Later works [13, 14] extended this idea by allowing the input-dependent rate to include an unknown coefficient, with the control gain regulated through an adaptive law or a time-varying method to dominate the unknown coefficient. However, none of these approaches addressed the case where the input-dependent rate itself is entirely unknown.

When the input-dependent rate is unknown, designing a controller becomes significantly more challenging. For a constant rate, adaptive laws can estimate the constant, or time-varying parameters can be designed to regulate the gain into the stable margin [9, 15]. However, when the rate is both input-dependent and unknown, it becomes difficult to incorporate it into the design, making it hard to determine the appropriate control gain. Unlike the case of an input-dependent rate with an unknown coefficient [13, 14], where partial information helps in evaluating the control gain feasibility, this scenario offers no such information. This situation also differs from our previous work on state feedback control for unknown input-dependent rates [16], which used a value function to update the control gain. In the output feedback case, however, designing such a value function is not feasible.

This paper presents a novel method to solve the output feedback regulation problem for the feedforward nonlinear system whose nonlinear terms have an unknown input-dependent rate. Unlike existing methods, we first establish an upper bound for the input, transforming the input-dependent rate into a condition with an unknown constant. Then, a time-varying parameter is employed to regulate the system within a stable margin and ensure that the input remains bounded. Finally, we present a method to estimate the control gain through the adaptive law. Our contributions can be summarized as follows:

- Our approach fundamentally differs from existing dynamic gain methods [12–14]. While previous methods designed a dynamic parameter larger than the input-dependent incremental rate, we regulate the input within a predefined bounded set, allowing the control parameter to be independent of the input-dependent incremental rate.
- We relax the conditions on nonlinear terms compared to existing works [13, 17]. Specifically, we address incremental rates that depend on both unknown input-dependent dynamics and unknown time-varying logarithmic dynamics. Previous approaches typically assumed precise knowledge of input-dependent dynamics, which was critical for designing control gains.

The remainder of this paper is organized as follows: We describe the problem in Section 2. Then, the novel method is introduced in Section 3 to respectively design a dynamic time-varying controller and an adaptive controller. Two examples are considered to illustrate the effectiveness of the proposed methods in Section 4. Some ending remarks are summarized in Section 5, while a reference list ends this paper.

Notation: We employ $\|\cdot\|$ to denote the Euclidean norm for vectors or the induced Euclidean norm for matrices. For matrix P, $P^\top$ represents its transpose, and $\lambda_{\max}(P)$, $\lambda_{\min}(P)$ denote the largest eigenvalue and the smallest eigenvalue of the matrix P, respectively. We use I to denote an $n \times n$ identity matrix.

2 | Problem Formulation

Consider the feedforward nonlinear system

$$\begin{aligned} \dot{x}_i(t) &= x_{i+1}(t) + f_i(t, x(t), u(t)), \quad i = 1, 2, \dots, n-1 \\ \dot{x}_n(t) &= u(t) \\ y(t) &= x_1(t) \end{aligned} \tag{1}$$

where $x = \left(x_1, x_2, \dots, x_n\right)^\top \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}$ is the system input, and $y \in \mathbb{R}$ is the system output. Without loss of generality, we assume that the initial instant is t_0, and the initial state is $x(t_0) \in \mathbb{R}^n$. The functions f_1 to f_{n-1} are continuous and satisfy the following assumption.

Assumption 1. For any $x = \left(x_1, x_2, \dots, x_n\right)^\top \in \mathbb{R}^n$, $u \in \mathbb{R}$, it holds

$$\begin{gathered} \left|f_i(t, x, u)\right| \leq \ln(c_1 t + c_2)\phi(u)\left(\left|x_{i+2}\right| + \cdots + \left|x_n\right| + |u|\right) \\ i = 1, 2, \dots, n-1 \end{gathered} \tag{2}$$

where $c_1 \geq 0$, $c_2 \geq 1 - c_1 t_0$ are unknown constants, and $\phi(u)$ is an unknown continuous function with respect to u.

Remark 1. System (1) exhibits an upper triangular structure, where the nonlinearity depends not only on the unmeasured states but also on the control input. Assumption 1 plays a crucial role in addressing the output feedback control problem for feedforward nonlinear systems. The growth rate $\ln(c_1 t + c_2)\phi(u)$ is nonlinear and entirely unknown. A summary of the conditions considered in the existing literature is presented in Table 1. From this comparison, it is clear that Assumption 1 offers a much more relaxed condition than those found in previous studies.

Our objective is to design an output feedback controller

$$u(t) = h(t, z(t)), \quad \dot{z}(t) = g(t, y(t), z(t)) \tag{3}$$

which can globally regulate the state of system (1) to the equilibrium point $x = 0$, $z = 0$. That is, for any initial state $x(t_0) \in \mathbb{R}^n$, $z(t_0) \in \mathbb{R}^n$, the state of the closed-loop system (1), (3) satisfy

$$\lim_{t \to +\infty} x(t) = 0, \quad \lim_{t \to +\infty} z(t) = 0$$

Significant progress has been made in addressing global regulation for feedforward nonlinear systems. In the context of low-gain feedback control design, the control gain can be characterized by a parameter, which should be sufficiently large relative to the incremental rate of the nonlinearity. This approach has been refined in various forms, including the dynamic form [12], adaptive form [13, 14], and time-varying form [15, 17]. In this paper, we respectively adopt the time-varying method and the adaptive method to design the control gain, leveraging the following lemma, which can be readily derived using established analyses [12, 21].

Lemma 1. *Let*

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \tag{4}$$

$C = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \end{pmatrix}$ *and* $D = \mathrm{diag}(n, n-1, \dots, 1)$. *Then, there exists a row vector* K, *a column vector* L, *a positive defined matrix* $P = \mathrm{diag}\{P_1, P_2\}$, *and a positive constant* h *such that*

$$\mathcal{A}^\top P + P\mathcal{A} \leq -I, \quad P\mathcal{D} + \mathcal{D}P \geq hI \tag{5}$$

TABLE 1 | Existing works on the growth rate for feedforward nonlinear systems via output feedback control. (θ represents an unknown constant and c is a known constant.)

	Constant	Function	Time-varying
Known	c [5, 18–20]	$\phi(u)$ [12, 21]	\
Parameter unknown	\	$\theta\phi(u)$ [13]	$\theta(1+t^c)$ [17], $\ln(\theta_1 t+\theta_2)$ [9]
Entirely unknown	θ [15, 22]	This work	\

where

$$\mathcal{A}=\begin{pmatrix} A-LC & 0 \\ -LC & A-BK \end{pmatrix},\quad \mathcal{D}=\begin{pmatrix} D & 0 \\ 0 & D \end{pmatrix} \tag{6}$$

3 | Main Results

3.1 | Design of the Time-Varying Control

Before we design, we define $g_\alpha(s)$ is a function satisfying the following conditions:

1. $g_\alpha(s)$ is continuous with respect to the variable $s\in\mathbb{R}$, and bounded by α, that is, $|g_\alpha(s)|\le\alpha$
2. $\beta(s):=\left|\frac{g_\alpha(s)-s}{s}\right|$ is continuous with respect to the variable $s\in\mathbb{R}$, and $\lim_{s\to 0}\beta(s)=0$.

Remark 2. Such a continuous function $g_\alpha(s)$ can be $g_\alpha(s)=\frac{\alpha s}{\alpha+|s|}, \alpha>0$ or $g_\alpha(s)=\alpha\cdot\tanh\left(\frac{s}{\alpha}\right), \alpha>0$. For $g_\alpha(s)=\frac{\alpha s}{\alpha+|s|}$, we get

$$\beta(s)=\frac{|s|}{\alpha+|s|},\quad \lim_{s\to+\infty}\beta(s)=0$$

For $g_\alpha(s)=\alpha\cdot\tanh\left(\frac{s}{\alpha}\right)$, we get

$$\beta(s)=\left|\frac{s^2}{3\alpha^2}-\frac{2s^4}{15\alpha^4}+\frac{17s^6}{315\alpha^6}-\dots\right|,\quad |s|<\frac{\alpha\pi}{2}$$

and it can be deduced into $\lim_{s\to+\infty}\beta(s)=0$. Thus, for both functions, our needed conditions are meet.

Employing the design of $K=(k_1,k_2,\dots,k_n)$, $L=(l_1,l_2,\dots,l_n)^\top$, matrix P, and constant h from Lemma 1, we design the controller (3) as

$$u(t)=g_\alpha(v(t)),\qquad t\ge t_0 \tag{7}$$

The dynamic $v(t)$ is given as

$$v(t)=-\frac{k_1}{r^n(t)}z_1(t)-\frac{k_2}{r^{n-1}(t)}z_2(t)-\dots-\frac{k_n}{r(t)}z_n(t) \tag{8}$$

where $r(t)\ge 1$ is a time-varying dynamic designed as

$$\dot r(t)=\frac{2}{h}\beta(v(t))\|K\|\|P\|+\frac{1}{h}\frac{1}{r(t)}\sqrt{n}\beta^2(v(t))\|K\|\|P\|+q$$
$$r(t_0)\ge 1 \tag{9}$$

with q being a positive constant satisfying $q^{-1}>8n\lambda_{\max}(P)$. The variable $z=(z_1,z_2,\dots,z_n)^\top$ is the state of the dynamic

$$\dot z_i(t)=z_{i+1}(t)-\frac{l_i}{r^i(t)}(z_1(t)-y(t)),\quad i=1,2,\dots,n-1$$
$$\dot z_n(t)=v(t)-\frac{l_n}{r^n(t)}(z_1(t)-y(t)) \tag{10}$$

It is worth noting that the design of $u(t)$ in (7) differs significantly from the existing designs. This approach specifically ensures that $u(t)$ remains bounded, which can further simplify the condition (2). From the properties of $g_\alpha(\cdot)$, it holds $|u(t)|\le\alpha$ for any $v(t)\in\mathbb{R}$. Noted that $\phi(u)$ in (2) is continuous with respect to u, and we can simplify Assumption 1 as

$$|f_i(t,x,u)|\le\theta\ln(c_1t+c_2)\left(|x_{i+2}|+\dots+|x_{n+1}|\right) \tag{11}$$

where θ is an unknown constant. This unknown constant θ is dependent on the unknown dynamic $\phi(u)$ and the parameter α.

This approach causes the parameter r to differ from that in existing methods. It regulates the control gain, which governs not only the nonlinearity but also the effects of the bounded input. Thus, it consists of two parts. The first part is

$$\dot r(t)\ge\frac{2}{h}\beta(v(t))\|K\|\|P\|+\frac{1}{h}\frac{1}{r(t)}\sqrt{n}\beta^2(v(t))\|K\|\|P\|$$

which we employed to regulate the error $v(t)-u(t)$. The other is

$$\dot r(t)\ge q>0$$

which is utilized to dominate the unknown constant. The main result is summarized as below:

Theorem 1. *Under Assumption 1, global regulation of system (1) can be achieved through the controller (7–10).*

Proof. For the convenience of the readers, we break up the proof into three parts. We first introduce an auxiliary variable $Z(t)$. Then, we prove that the variable $Z(t)$ is converging to 0. Finally, we go back to the original state $z(t)$, $x(t)$, and we proved that $z(t)\to 0$, $x(t)\to 0$ as $t\to 0$.

Part I: Constructing auxiliary variable $Z(t)=\left(\xi^\top(t),\eta^\top(t)\right)^\top$. We first consider $\xi(t)=(\xi_1(t),\dots,\xi_n(t))^\top$, where

$$\xi_i(t)=\frac{z_i(t)-x_i(t)}{r^{n+1-i}(t)},\quad i=1,2,\dots,n$$

From (1), (10), we obtain

$$\dot\xi(t)=\frac{1}{r(t)}(A-LC)\xi(t)-\frac{\dot r(t)}{r(t)}D\xi(t)+\frac{1}{r(t)}G_1(t)-G_2(t) \tag{12}$$

where matrices A, C, D are the same with the denotation in Lemma 1, $G_1(t) = (0, 0, \dots, 0, v(t) - u(t))^\top$, and

$$G_2(t) = \begin{pmatrix} \frac{1}{r^n(t)} f_1(t, x(t), u(t)) \\ \frac{1}{r^{n-1}(t)} f_2(t, x(t), u(t)) \\ \vdots \\ \frac{1}{r^2(t)} f_{n-1}(t, x(t), u(t)) \\ 0 \end{pmatrix}$$

Another variable $\eta(t) = \left(\eta_1(t), \dots, \eta_n(t)\right)^\top$ is given as

$$\eta_i(t) = \frac{z_i(t)}{r^{n+1-i}(t)}, \quad i = 1, 2, \dots, n$$

Then, the dynamic $v(t)$ in (8) is expressed as

$$v(t) = -K\eta(t)$$

and from (10), we get

$$\dot{\eta}(t) = \frac{1}{r(t)}(A - BK)\eta(t) - \frac{1}{r(t)} LC\xi(t) - \frac{\dot{r}(t)}{r(t)} D\eta(t) \tag{13}$$

where B is denoted as (4).

Putting (12) and (13) together, we get the dynamic of $Z(t)$ as

$$\dot{z}(t) = \frac{1}{r(t)}\mathcal{A}Z(t) - \frac{\dot{r}(t)}{r(t)}\mathcal{D}Z(t) + \frac{1}{r(t)}\mathcal{G}_1(t) + \mathcal{G}_2(t) \tag{14}$$

where $\mathcal{A}$, $\mathcal{D}$ are denoted as (6), $\mathcal{G}_1(t) = (G_1^\top(t), 0^\top)^\top$, and $\mathcal{G}_2(t) = (-G_2^\top(t), 0^\top)^\top$.

Part II: Convergence of the variable $Z(t)$. Consider the function

$$V(t) = Z^\top(t)PZ(t)$$

where P is given in Lemma 1. Its derivative is computed as

$$\dot{V}(t) \leq -\frac{1}{r(t)}\|Z(t)\|^2 - h\frac{\dot{r}(t)}{r(t)}\|Z(t)\|^2 + \frac{2}{r(t)}Z^\top(t)P\mathcal{G}_1(t) + 2Z^\top(t)P\mathcal{G}_2(t) \tag{15}$$

where (5) is utilized.

Since

$$|u(t) - v(t)| = \beta(v(t))|v(t)| \leq \beta(v(t))\|K\|\|\eta(t)\|$$

we get

$$\frac{2}{r(t)}Z^\top(t)P\mathcal{G}_1(t) \leq \frac{2}{r(t)}\beta(v(t))\|K\|\|P\|\|Z(t)\|^2 \tag{16}$$

On the other hand, because the nonlinear terms $f_i(t, x(t), u(t))$ satisfy (11) and $r(t) \geq 1$, we obtain

$$\frac{1}{r^{n+1-i}(t)}|f_i(t, x(t), u(t))| \leq \frac{\theta \ln(c_1 t + c_2)}{r^2(t)}\left(\sqrt{2n}\|Z(t)\| + |u(t)|\right)$$

Because

$$|u(t)| \leq |u(t) - v(t)| + |v(t)| \leq (\beta(v(t)) + 1)\|K\|\|\eta(t)\|$$

we get

$$\frac{1}{r^{n+1-i}(t)}|f_i(t, x(t), u(t))| \leq \frac{\theta \ln(c_1 t + c_2)}{r^2(t)}(\beta(v(t)) + \sqrt{2n} + 1)\|K\|\|Z(t)\|$$

Thus, we get

$$\|\mathcal{G}_2(t)\| \leq \frac{\theta\sqrt{n}\ln(c_1 t + c_2)}{r^2(t)}(\beta(v(t)) + \sqrt{2n} + 1)\|K\|\|Z(t)\|$$

and

$$\begin{aligned} 2Z^\top(t)P\mathcal{G}_2(t) \leq{}& \frac{2\theta\sqrt{n}\ln(c_1 t + c_2)}{r^2(t)} \\ &\times (\beta(v(t)) + \sqrt{2n} + 1)\|K\|\|P\|\|Z(t)\|^2 \\ \leq{}& \frac{2\theta \ln(c_1 t + c_2)(\sqrt{2n} + 1) + \theta^2\ln^2(c_1 t + c_2)}{r^2(t)} \\ &\times \sqrt{n}\|K\|\|P\|\|Z(t)\|^2 + \frac{1}{r^2(t)}\beta^2(v(t)) \\ &\times \sqrt{n}\|K\|\|P\|\|Z(t)\|^2 \end{aligned} \tag{17}$$

Based on (9), (16), and (17), we update (15) as

$$\begin{aligned} \dot{V}(t) \leq{}& -\frac{1}{r(t)}\|Z(t)\|^2 \\ &+ \frac{2\theta \ln(c_1 t + c_2)(\sqrt{2n} + 1) + \theta^2\ln^2(c_1 t + c_2)}{r^2(t)} \\ &\times \sqrt{n}\|K\|\|P\|\|Z(t)\|^2 \end{aligned}$$

From (9), we ensure that

$$\lim_{t \to +\infty} \frac{2\theta \ln(c_1 t + c_2)(\sqrt{2n} + 1) + \theta^2\ln^2(c_1 t + c_2)}{r(t)} \times \sqrt{n}\|K\|\|P\| = 0$$

holds for any constants $\theta \geq 0$, $c_1 \geq 0$, and $c_2 \geq 1 - c_1 t_0$. Then, there exists an instant $t_1 \geq t_0$ such that

$$\frac{2\theta \ln(c_1 t + c_2)(\sqrt{2n} + 1) + \theta^2\ln^2(c_1 t + c_2)}{r(t)} \times \sqrt{n}\|K\|\|P\| \leq \frac{1}{2}, \quad t \geq t_1$$

In this case, we get

$$\dot{V}(t) \leq -\frac{1}{2r(t)}\|Z(t)\|^2 \leq -\frac{1}{2\lambda_{\max}(P)r(t)}V(t), \quad t \geq t_1 \tag{18}$$

Because $r(t) \to +\infty$ as $t \to +\infty$, we cannot directly get the convergence of $Z(t)$ from the above inequality. We further analyze as fellow. Noticing that $\dot{V}(t) \leq 0$, we ensure $v(t) = -K\eta(t)$ to be bounded. Since $\beta(s)$ is continuous with respect to s, we can ensure $\beta(v(t))$ to be bounded. Back to (9), there exists a constant $\bar{r}$ to meet

$\dot{r}(t) \le \bar{r}$. Then, it holds $r(t) \le \bar{r} \cdot (t - t_1) + r(t_1)$, $t \ge t_1$. From (19), we arrive at

$$V(t) \le \left(\frac{r(t_1)}{\bar{r} \cdot (t - t_1) + r(t_1)} \right)^{\frac{1}{2\lambda_{\max}(P)\bar{r}}} V(t_1)$$

which means

$$\lim_{t \to +\infty} V(t) = 0$$

Because $V(t) \ge \lambda_{\min}(P)\|Z(t)\|^2$, we achieve

$$\lim_{t \to +\infty} Z(t) = 0 \tag{19}$$

Noticed that since $r(t)$ is time-varying, we cannot directly get the convergence of the system states from (19). The following analysis is needed.

Part III: Convergence of the original states $z(t)$, $x(t)$. Since

$$|z_i(t)| \le r^n(t)|\eta_i(t)|, \quad |x_i| \le r^n(t)\big(|\eta_i(t)| + |\xi_i(t)|\big)$$

hold for any $i = 1, 2, \ldots, n$, we can get the convergence of $z(t)$, $x(t)$ through considering $\tilde{V}(t) = r^{2n}(t)V(t)$. From (18), we get

$$\dot{\tilde{V}}(t) \le -\frac{1}{2\lambda_{\max}(P)} \frac{1}{r(t)} \tilde{V}(t) + 2n \frac{\dot{r}(t)}{r(t)} \tilde{V}(t), t \ge t_1$$

We obtain that when $t \to +\infty$, it holds $Z(t) \to 0$, $\upsilon(t) \to 0$, $\beta(\upsilon(t)) \to 0$. Then, there exists an instant $t_2 \ge t_1$ such that

$$\dot{r}(t) \le \frac{1}{2n} \frac{1}{4\lambda_{\max}(P)} + q, \quad t \ge t_2$$

Thus, we arrive at

$$\dot{\tilde{V}}(t) \le -\left(\frac{1}{4\lambda_{\max}(P)} - 2nq \right) \frac{1}{r(t)} \tilde{V}(t), t \ge t_2$$

From the design of q, we get $\left(\frac{1}{4\lambda_{\max}(P)} - 2nq \right) > 0$. Similar to the above analysis, we can get

$$\lim_{t \to +\infty} \tilde{V}(t) = 0$$

which indicates the convergence of the states $z(t)$, $x(t)$. This ends the proof. □

Remark 3. The system performance of our controller is heavily affected by the constant α. When α is small, the nonlinear growth rate may be small, and the bound of control input is small. We use the constant α to transform Assumption 1 into an unknown time-varying incremental rate $\theta \ln(c_1 t + c_2)$ as shown in (11). Although the time-varying parameter is introduced, $r(t)$ is regulated to dominate the unknown growth rate. Thus, by carefully designing the constant α, the transient performance of the system can be improved.

3.2 | Design of the Adaptive Control

In the control design of (7–10), the parameter α tends to ∞, which is reasonable since condition (2) includes the time-varying coefficient $\ln(c_1 t + c_2)$. When the time-varying coefficient is absent, we generally design a finite control gain. In the following, we introduce an alternative approach to control design.

Theorem 2. *Consider system (1) whose nonlinear terms satisfy*

$$|f_i(t, x, u)| \le \phi(u)\big(|x_{i+2}| + \ldots + |x_n| + |u|\big), \quad i = 1, 2, \ldots, n-1$$

with $\phi(u)$ being an unknown continuous function. Let the controller be (7), (8), (10). Then, by designing the dynamic parameter $r(t)$ as

$$\dot{r}(t) = \frac{2}{h}\beta(\upsilon(t))\|K\|\|P\| + \frac{1}{h}\frac{1}{r(t)}\sqrt{n}\beta^2(\upsilon(t))\|K\|\|P\| + \sum_{i=1}^{n} \frac{1}{r^{2n+3-2i}} z_i^2(t) + \frac{1}{r^{2n+1}(t)}\big(y(t) - z_1(t)\big)^2$$

we can achieve

$$\lim_{t \to +\infty} \|z(t)\| = 0, \qquad \lim_{t \to +\infty} \|x(t)\| = 0$$

Proof. By considering the auxiliary variable $Z(t)$, we also get (14). Since $|u(t)| \le \alpha$, we can express the condition as

$$|f_i(t, x, u)| \le \theta\big(|x_{i+2}| + \cdots + |x_n| + |u|\big)$$

where θ is an unknown constant.

Then, by considering the Lyapunov function $V(t) = Z^\top(t)PZ(t)$, we get its derivative as

$$\dot{V}(t) \le -\frac{1}{r(t)}\|Z(t)\|^2 + \frac{\Theta}{r^2(t)}\|Z(t)\|^2$$

where $\Theta = \left(2\theta(\sqrt{2n} + 1) + \theta^2\right)\sqrt{n}\|K\|\|P\|$.

The following analysis is divided into two cases:

Case I: there is an instant t_1 such that $r(t_1) \ge \max\{2\Theta, 1\}$. Since $r(t)$ is a non-decreasing function, we get $r(t) \ge r(t_1), t \ge t_1$. Then, it holds

$$\dot{V}(t) \le -\frac{1}{2r(t)}\|Z(t)\|^2 \le -\frac{1}{2\lambda_{\max}(P)r(t)}V(t), \quad t \ge t_1$$

Integrating both sides from t_1 to t, we get

$$V(t_1) \ge V(t_1) - V(t) \ge \int_{t_1}^{t} \frac{1}{2\lambda_{\max}(P)r(s)} V(s)ds$$

and

$$r(t) - r(t_1) \le \int_{t_1}^{t} \frac{2}{h}\beta(\upsilon(s))\|K\|\|P\|ds + \int_{t_1}^{t} \frac{1}{h}\frac{1}{r(s)}\sqrt{n}\beta^2(\upsilon(s))\|K\|\|P\|ds + \int_{t_1}^{t} \frac{1}{\lambda_{\min}(P)r(s)} V(s)ds$$

$$\leq \int_{t_1}^{t} \frac{2}{h}\beta(v(s))\|K\|\|P\|ds + \int_{t_1}^{t} \frac{1}{h}\frac{1}{r(s)}\sqrt{n}\beta^2(v(s))\|K\|\|P\|ds + \frac{2\lambda_{\max}(P)}{\lambda_{\min}(P)}V(t_1)$$

where $V(s) \leq V(t_1), s \in [t_1, t)$ is employed. Meanwhile, it holds

$$|v(t)| \leq \|K\|\|z(t)\| \leq \|K\|\|Z(t)\| \leq \|K\|\sqrt{\frac{1}{\lambda_{\min}(P)}V(t)} \leq \|K\|\sqrt{\frac{1}{\lambda_{\min}(P)}V(t_1)}$$

Noted that the above inequality is holds for any $t \in [t_1, +\infty)$. Thus, $v(t)$ is bounded on $[t_1, +\infty)$. Since $\beta(v(s))$ is continuous, we can find a constant $\bar{r}$ such that $r(t) \leq \bar{r}(t - t_1) + r(t_1)$. The following analysis is similar to the proof for Theorem 1, and we omit it here.

Case II: the upper bound of $r(t)$ is smaller than $\max\{2\Theta, 1\}$. Since

$$r\dot{r} \geq \|\eta(t)\|^2 + \frac{1}{r^{2n}(t)}\left(y(t) - z_1(t)\right)^2$$

we get

$$\int_{t_0}^{t} \|\eta(s)\|^2 ds + \int_{t_0}^{t} \frac{1}{r^{2n}(s)}\left(y(s) - z_1(s)\right)^2 ds < r^2(t) < \max\{4\Theta^2, 1\} \tag{20}$$

Thus, it holds

$$\lim_{t\to+\infty} \|\eta(t)\| = 0, \qquad \lim_{t\to+\infty} \frac{1}{r^{2n}(t)}(y(t) - z_1(t))^2 = 0$$

which means $v(t) = K\eta(t)$ is bounded. That is, there exists a constant $\bar{\beta}$ such that $\beta(v(t)) \leq \bar{\beta}$. In the follow, We prove that $\lim_{t\to+\infty} \|x(t)\| = 0$.

Consider another auxiliary variable $\varepsilon(t) = \left(\varepsilon_1(t), \ldots, \varepsilon_n(t)\right)^\top$ where

$$\varepsilon_i(t) = \frac{z_i(t) - x_i(t)}{\gamma^{n+1-i}(t)}, \quad i = 1, 2, \ldots, n$$

with $\gamma(t) = cr(t)$. Here, $c \geq 1$ is a constant that is not utilized in the control design.

Then, we get

$$\dot{\varepsilon}(t) = \frac{1}{\gamma(t)}(A - LC)\varepsilon(t) - \frac{1}{\gamma(t)}L\Gamma\varepsilon_1(t) - \frac{\dot{\gamma}(t)}{\gamma(t)}D\varepsilon(t) + \frac{1}{\gamma(t)}G_1(t) - \tilde{G}_2(t)$$

where A, C, D, G_1 are the same with (12), $\Gamma = \mathrm{diag}\{c^n, c^{n-1}, \ldots, c\}$ and

$$\tilde{G}_2(t) = \begin{pmatrix} \frac{1}{\gamma^n(t)}f_1(t, x(t), u(t)) \\ \frac{1}{\gamma^{n-1}(t)}f_2(t, x(t), u(t)) \\ \vdots \\ \frac{1}{\gamma^2(t)}f_{n-1}(t, x(t), u(t)) \\ 0 \end{pmatrix}$$

Consider $V_1(t) = \varepsilon^\top(t)P_1\varepsilon(t)$, and its derivative is computed as

$$\dot{V}_1(t) \leq -\frac{1}{\gamma(t)}\|\varepsilon(t)\|^2 - \frac{2}{\gamma(t)}\varepsilon^\top(t)P_1 L\Gamma\varepsilon_1 + \frac{2}{\gamma(t)}\varepsilon^\top(t)P_1 G_1(t) - \frac{2}{\gamma(t)}\varepsilon^\top(t)P_1\tilde{G}_2(t) - h\frac{\dot{\gamma}(t)}{\gamma(t)}\|\varepsilon(t)\|^2 \tag{21}$$

In the following, we estimate each terms. Begin with the second term on the right hand; we get

$$-\frac{2}{\gamma(t)}\varepsilon^\top(t)P_1 L\Gamma\varepsilon_1(t) \leq \frac{1}{4\gamma(t)}\|\varepsilon(t)\|^2 + \frac{m_1}{\gamma(t)}\varepsilon_1^2(t) \tag{22}$$

where $m_1 = 4\|P_1 L\Gamma\|^2$. Following the same analysis in (16), we also can get the estimation

$$\frac{2}{\gamma(t)}\varepsilon^\top(t)P_1 G_1(t) \leq \frac{2}{\gamma(t)}\beta(v(t))\|KP_1\|\|\varepsilon(t)\|\|\eta(t)\| \leq \frac{1}{4\gamma(t)}\|\varepsilon(t)\|^2 + \frac{m_2}{\gamma(t)}\|\eta(t)\|^2 \tag{23}$$

where $m_2 = 4\bar{\beta}^2\|KP_1\|^2$. For the fourth term, we obtain

$$2\varepsilon^\top(t)P_1\tilde{G}_2(t) \leq \frac{2}{\gamma^2(t)}\theta\|P_1\|\|\varepsilon(t)\| \times (n\|\varepsilon(t)\| + (n + \|K\|\sqrt{n})\|\eta(t)\|) \leq \frac{1}{\gamma^2(t)}\theta\|P_1\|(2n+1)\|\varepsilon(t)\|^2 + \frac{1}{\gamma^2(t)}\theta\|P_1\|(n + \|K\|\sqrt{n})^2\|\eta(t)\|^2 \tag{24}$$

Substituting (22–24) into (21), we get

$$\dot{V}_1(t) \leq -\frac{1}{2\gamma(t)}\|\varepsilon(t)\|^2 + \frac{m_1}{\gamma(t)}\varepsilon_1^2(t) + \frac{m_2}{\gamma(t)}\|\eta(t)\|^2 + \frac{1}{\gamma^2(t)}\theta\|P_1\|(2n+1)\|\varepsilon(t)\|^2 + \frac{1}{\gamma^2(t)}\theta\|P_1\|(n + \|K\|\sqrt{n})^2\|\eta(t)\|^2$$

When $c \geq \max\{4\theta\|P_1\|(2n+1), 1\}$, we get

$$\dot{V}_1(t) \leq -\frac{1}{4cr(t)}\|\varepsilon(t)\|^2 + \frac{m_1}{\gamma(t)}\varepsilon_1^2(t) + \frac{m_2}{\gamma(t)}\|\eta(t)\|^2 + \frac{1}{\gamma^2(t)}\theta\|P_1\|(n + \|K\|\sqrt{n})^2\|\eta(t)\|^2$$
$$\leq -M_1V_1(t) + M_2\frac{1}{r^{2n+1}(t)}\left(y(t) - z_1(t)\right)^2 + M_2\sum_{i=1}^{n}\frac{1}{r^{2n+3-2i}(s)}z_i^2(t)$$

where $M_1 = \frac{1}{16c\Theta\|P\|}\frac{1}{\lambda_{\max}(P_1)}$, and $M_2 = \max\{m_1, m_2 + \theta\|P_1\|(n + \|K\|\sqrt{n})^2$. Then, it holds

$$V_1(t) \leq e^{-M_1(t-t_0)}V_1(t_0) + M_2\int_{t_0}^{t} e^{-M_1(t-s)} \times \left(\frac{1}{r^{2n+1}(s)}\left(y(s) - z_1(s)\right)^2 + \sum_{i=1}^{n}\frac{1}{r^{2n+3-2i}(s)}z_i^2(s)\right)ds \tag{25}$$

From (20), we get

$$\lim_{t\to+\infty}\int_{\frac{t}{2}}^{t}\left(\frac{1}{r^{2n+1}(s)}\left(y(s)-z_1(s)\right)^2+\sum_{i=1}^{n}\frac{1}{r^{2n+3-2i}(s)}z_i^2(s)\right)ds=0$$

and there exists a positive constant ρ such that

$$\frac{1}{r^{2n+1}(t)}\left(y(t)-z_1(t)\right)^2+\sum_{i=1}^{n}\frac{1}{r^{2n+3-2i}(t)}z_i^2(t)\le\rho,\quad \forall t\in[t_0,+\infty)$$

Then, (25) can be expressed as

$$\begin{aligned}V_1(t)\le{}& e^{-M_1(t-t_0)}V_1(t_0)+M_2\int_{t_0}^{\frac{t}{2}}e^{-M_1(t-s)}\\&\times\left(\frac{1}{r^{2n+1}(s)}\left(y(s)-z_1(s)\right)^2+\sum_{i=1}^{n}\frac{1}{r^{2n+3-2i}(s)}z_i^2(s)\right)ds\\&+M_2\int_{\frac{t}{2}}^{t}e^{-M_1(t-s)}\\&\times\left(\frac{1}{r^{2n+1}(s)}\left(y(s)-z_1(s)\right)^2+\sum_{i=1}^{n}\frac{1}{r^{2n+3-2i}(s)}z_i^2(s)\right)ds\\\le{}& e^{-M_1(t-t_0)}V_1(t_0)+M_2\rho\int_{t_0}^{\frac{t}{2}}e^{-M_1(t-s)}ds\\&+M_2\int_{\frac{t}{2}}^{t}\left(\frac{1}{r^{2n+1}(s)}\left(y(s)-z_1(s)\right)^2\right.\\&\left.+\sum_{i=1}^{n}\frac{1}{r^{2n+3-2i}(s)}z_i^2(s)\right)ds\end{aligned}$$

Since

$$\int_{t_0}^{t}e^{-M_1(t-s)}ds=\frac{1}{M_1}\left(e^{-M_1\frac{t}{2}}-e^{-M_1(t-t_0)}\right)$$

we get $\lim_{t\to+\infty}V_1(t)=0$. Then, it holds $\lim_{t\to+\infty}\|x(t)\|=\lim_{t\to+\infty}\|x(t)-z(t)\|+\lim_{t\to+\infty}\|z(t)\|=0$.

Therefore, considering both cases, we conclude that $\lim_{t\to+\infty}\|x(t)\|=0$ and $\lim_{t\to+\infty}\|z(t)\|=0$. This ends the proof. □

Remark 4. When all the states are available to the control design, the time-varying state feedback controller or the adaptive state feedback controller can be designed. In this case, the dynamic $v(t)$ in (8) is changed as

$$v(t)=-\frac{k_1}{r^n(t)}x_1(t)-\frac{k_2}{r^{n-1}(t)}z_2(t)-\cdots-\frac{k_n}{r(t)}z_n(t)$$

and the dynamic $r(t)$ can be designed to solve the state feedback stabilization problem. The performance analysis is similar to that in this paper, which is omitted here.

4 | Simulations

4.1 | Example 1

Consider the nonlinear liquid level control resonant circuit system [11, 13]

$$\begin{cases}\dot{\mathbf{i}}_{L_1}(t)=-\frac{\mathbf{v}_c(t)}{L_1}-\frac{R_a\left(\mathbf{i}_{L_2}(t)-0.5\sin(\mathbf{v}_c(t))\right)}{L_1}\\\dot{\mathbf{v}}_c(t)=\frac{\mathbf{i}_{L_2}(t)}{C}-\frac{0.5\sin(\mathbf{v}_c(t))}{C}\\\dot{\mathbf{i}}_{L_2}(t)=-\frac{R_b\mathbf{i}_{L_2}(t)}{L_2}+\frac{\mathbf{v}(t)}{L_2}\end{cases}$$

where $\mathbf{v}(t)$ is a control input voltage, $\mathbf{v}_c(t)$ is the voltage across the capacitor C, $\mathbf{i}_{L_1}$ and $\mathbf{i}_{L_2}$ are the currents through the tunnel diode, R, R_a, and R_b are the resistances, and L_1 and L_2 are the inductors.

Let the state $x_1(t)=L_1\mathbf{i}_{L_1}(t)$, $x_2(t)=-\mathbf{v}_c(t)$, and $x_3=-\frac{1}{C}\left(\mathbf{i}_{L_2}(t)-0.5\sin(\mathbf{v}_c(t))\right)$. Choosing the pre-feedback control input as $\mathbf{v}(t)=-L_2Cu-R_bCx_3-0.5R_b\sin(x_2)-0.5L_2\cos(x_2)x_3$. One supposes that $R_a=\ln(c_1t+c_2)u(t)$ with c_1, c_2 being unknown positive constants. $L_1=L_2=1$, $C=10$, and $R_b=1$. Then, we have the following system model

$$\begin{cases}\dot{x}_1(t)=x_2(t)+10\ln(c_1t+c_2)u(t)x_3(t)\\\dot{x}_2(t)=x_3(t)\\\dot{x}_3(t)=u(t)\\y(t)=x_1(t)\end{cases}\tag{26}$$

It is obvious that (26) satisfies Assumption 1. We assume the initial instant t_0 as 0. Following our dynamic time-varying method, we can choose $g_\alpha(s)=\frac{\alpha s}{\alpha+|s|}$, and design the controller as

$$u(t)=\frac{\alpha v(t)}{\alpha+|v(t)|}\tag{27}$$

where

$$v(t)=-\frac{z_1(t)}{r^3(t)}-3\frac{z_2(t)}{r^2(t)}-3\frac{z_3(t)}{r(t)}$$

$$\begin{cases}\dot{z}_1(t)=z_2(t)-3\frac{1}{r(t)}(z_1(t)-y(t))\\\dot{z}_2(t)=z_3(t)-3\frac{1}{r^2(t)}(z_1(t)-y(t))\\\dot{z}_3(t)=v(t)-\frac{1}{r^3(t)}(z_1(t)-y(t))\end{cases}\tag{28}$$

And, the time-varying parameter is designed as

$$\dot{r}(t)=\frac{65|v(t)|}{\alpha+|v(t)|}+\frac{65v^2(t)}{(\alpha+|v(t)|)^2r(t)}+10^{-6},\quad r(0)=1\tag{29}$$

By choosing the initial condition $x(0)=(0.5,-0.3,0.1)^\top$, $z(0)=(0,0,0)^\top$, we give the trajectory under different parameter α in Figure 1. It is shown that all the states are converging to the equilibrium $x=0$, $z=0$, which illustrates the effectiveness of our result. We can see from Figure 1a that the system is converging at about instant 250 s when $\alpha=1$. It has better performance in Figure 1d and e. But the system performance is deduced in Figure 1f, since the converging instant is larger than 40 s and the upper bound is larger than 2 s. Thus, the system performance can be optimized through choosing the parameter α.

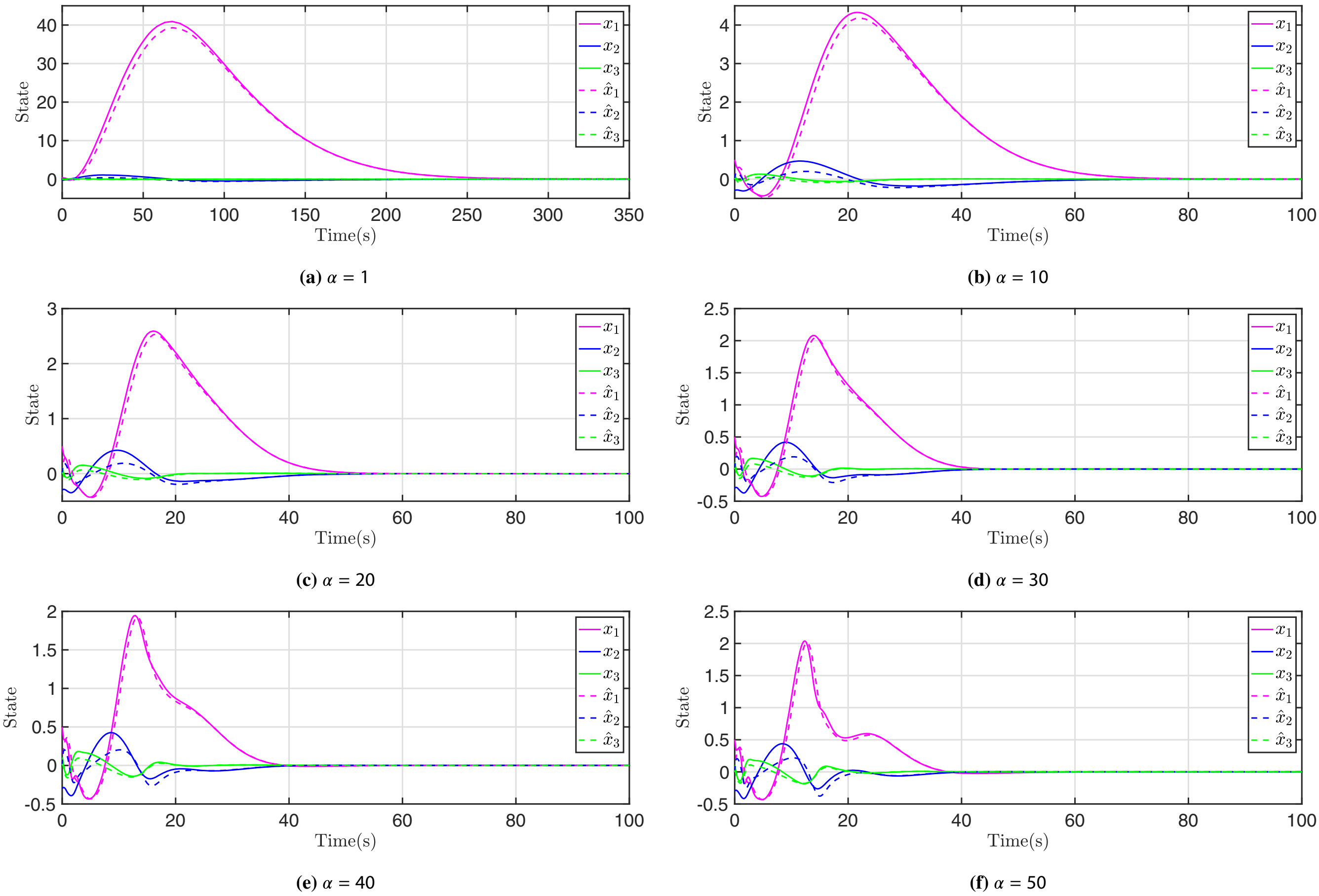

FIGURE 1 | The trajectory of the closed-loop system (26–29) under different α. (a) $\alpha = 1$ (b) $\alpha = 10$ (c) $\alpha = 20$ (d) $\alpha = 30$ (e) $\alpha = 40$ (f) $\alpha = 50$.

4.2 | Example 2

We consider another example to illustrate the effectiveness of Theorem 2. Consider the nonlinear system

$$\begin{cases} \dot{x}_1 = x_2 + 10ux_3 \\ \dot{x}_2 = x_3 + 2u^2 \\ \dot{x}_3 = u \\ y = x_1 \end{cases} \tag{30}$$

where $x = \left(x_1, x_2, x_3\right)^\top$ is the system state, u is the system input, and y is the system output. Following Theorem 2, we design the control as

$$u(t) = g_\alpha(v(t)) \tag{31}$$

where

$$v(t) = -\frac{z_1(t)}{r^3(t)} - 3\frac{z_2(t)}{r^2(t)} - 3\frac{z_3(t)}{r(t)}$$

$$\begin{cases} \dot{z}_1(t) = z_2(t) - 3\frac{1}{r(t)}(z_1(t) - y(t)) \\ \dot{z}_2(t) = z_3(t) - 3\frac{1}{r^2(t)}(z_1(t) - y(t)) \\ \dot{z}_3(t) = v(t) - \frac{1}{r^3(t)}(z_1(t) - y(t)) \end{cases} \tag{32}$$

The adaptive parameter $r(t)$ is designed as

$$\dot{r}(t) = 65\beta(v(t)) + 65\beta^2(v(t))\frac{1}{r(t)} + \frac{z_1^2(t)}{r^7(t)} + \frac{z_2^2(t)}{r^5(t)} + \frac{z_3^2(t)}{r^3(t)} + \frac{\left(z_1(t) - y(t)\right)^2}{r^7(t)} \tag{33}$$

where $\beta(v(t)) = \left|\frac{g_\alpha(v(t)) - v(t)}{v(t)}\right|$.

We choose the initial condition $x(0) = (0.5, -0.3, 0.1)^\top$, $z(0) = (0, 0, 0)^\top$. When $g_\alpha(v) = \frac{\alpha v}{\alpha + |v|}$, we give the trajectory under different parameter α in Figure 2. We also consider $g_\alpha(v) = \alpha \cdot \tanh\left(\frac{v}{\alpha}\right)$, and the trajectory is given in Figure 3. It is shown that all the states are converging to the equilibrium $x = 0, z = 0$, which illustrates the effectiveness of our result. It is also shown that the system transient performance is dependent on the selecting parameter α.

5 | Conclusion

This paper proposes a novel method for designing a regulating controller for feedforward nonlinear systems, where the nonlinear terms have an unknown input-dependent rate. By introducing a bounded law, we simplified the complex incremental rate into a more manageable form. We then applied both time-varying

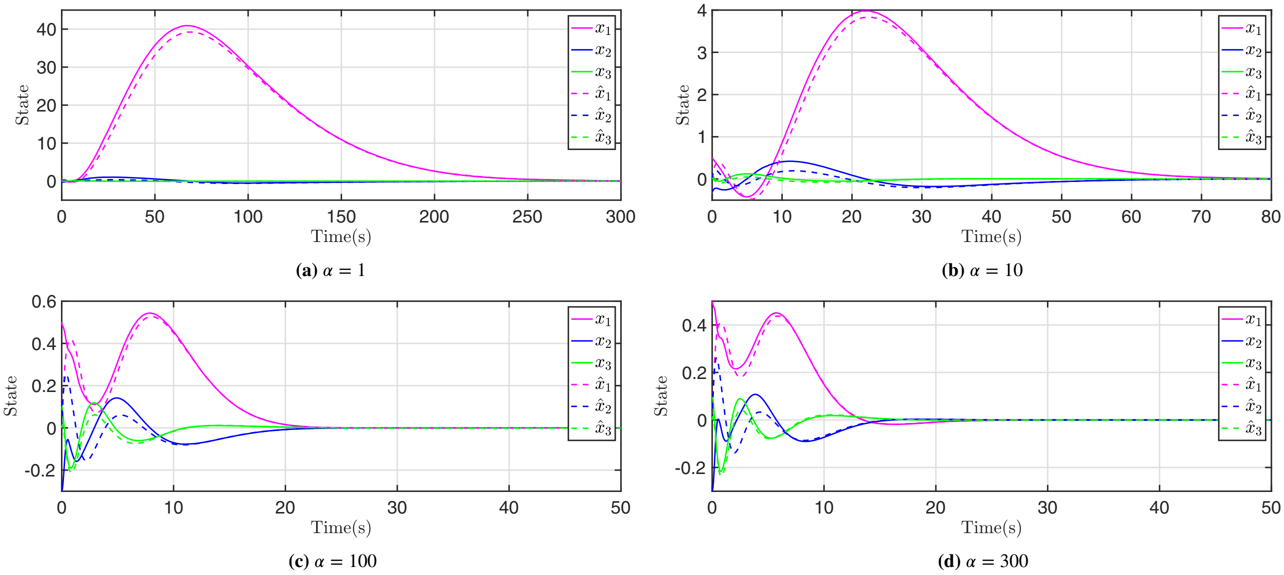

FIGURE 2 | The trajectory of the closed-loop system (30–33) with $g_\alpha(v) = \frac{\alpha v}{\alpha + |v|}$ under different α. (a) $\alpha = 1$ (b) $\alpha = 10$ (c) $\alpha = 100$ (d) $\alpha = 300$.

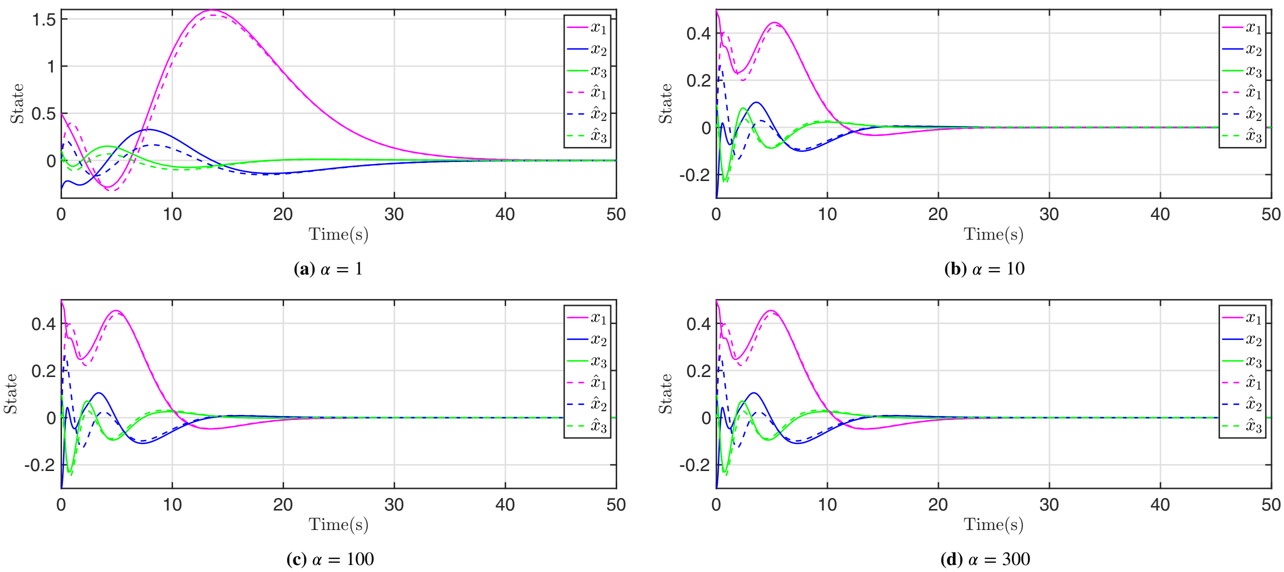

FIGURE 3 | The trajectory of the closed-loop system (30–33) with $g_\alpha(v) = \alpha \cdot \tanh\left(\frac{v}{\alpha}\right)$ under different α. (a) $\alpha = 1$ (b) $\alpha = 10$ (c) $\alpha = 100$ (d) $\alpha = 300$.

and adaptive methods to adjust the control gain. Since the control gain was designed independently of $\phi(u)$, we were able to modify its rate using a parameter α, potentially improving transient performance. Future research will focus on more complex scenarios, such as systems that include delays.

Conflicts of Interest

The authors declare no conflicts of interest.

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