Distributed State Estimation for Active Distribution Grids With Integrated Battery Energy Storage

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Abstract—This paper proposes a distributed state estimation (DSE) scheme for active distribution grids (ADGs) featuring integrated battery energy storage systems (BESS). Unlike traditional methods that treat electrical and storage states separately, the proposed approach utilizes a unified area-level model to capture the physical coupling between network voltages and BESS state-of-charge (SOC). We design a distributed cooperative estimator that combines local innovation correction with a consensus-based information fusion mechanism. Theoretical analysis proves the uniform ultimate boundedness of estimation errors under bounded disturbances. Simulation results on a modified IEEE 33-bus system demonstrate that the integration of storage dynamics and inter-area communication significantly improves estimation accuracy and regional consistency compared to localized estimation methods.

Index Terms—Active Distribution Grids; Distributed State Estimation; Battery Energy Storage Systems; Consensus Control; Cyber-Physical Systems

I. Introduction

With the increasing integration of distributed generation, flexible loads, and battery energy storage systems (BESSs), modern distribution networks are gradually evolving into active distribution grids (ADGs) with enhanced operational flexibility but also increased complexity [1]–[3]. Compared with conventional passive distribution networks, ADGs exhibit stronger temporal variability caused by renewable generation fluctuations, changing load demands, and storage charging/discharging behaviors [4]. In such systems, accurate state awareness is essential for secure operation, voltage regulation, and energy management.

State estimation has been widely used in power systems to provide reliable operating information for monitoring and control [5], [6]. However, state estimation for ADGs is more challenging than that for transmission systems. On the one hand, distribution networks usually have fewer measurement devices and lower measurement redundancy [7]. On the other hand, the operating conditions of ADGs become more dynamic due to the large-scale participation of distributed energy resources, flexible loads, and storage units [8]. These features make accurate state reconstruction difficult, especially when the system scale increases and the network operation becomes highly time-varying.

To address these issues, distributed state estimation has attracted increasing attention in recent years [9], [10]. By partitioning the network into multiple areas and assigning local estimators to different regions, distributed estimation can improve scalability and reduce the communication burden compared with centralized schemes. Moreover, this structure is consistent with the geographically distributed nature of ADGs, where local estimators process regional measurements and exchange information with neighboring estimators through communication networks [7], [11]. Related studies on distributed consensus, cooperative synchronization, and resilient leader–follower coordination have also shown the potential of distributed information interaction in networked dynamic systems [12]–[14]. These results provide useful motivation for developing distributed estimation schemes for ADGs.

The integration of BESSs further complicates the estimation problem [15]. The charging and discharging behaviors of storage units directly affect nodal power injections, voltage profiles, and power flows. Meanwhile, the operating condition of the distribution network also imposes constraints on storage operation. Therefore, BESSs should be incorporated into the state estimation model as dynamic components coupled with the electrical network, rather than being treated as isolated external devices [16]. This is particularly important for ADGs, where storage units play an increasingly important role in flexibility support, voltage regulation, and operational adjustment.

Motivated by the above observations, this paper studies distributed state estimation for active distribution grids with integrated battery energy storage. The overall ADG is modeled as a coupled large-scale dynamic system by stacking all regional states, including both electrical variables and storage-related states. Based on local measurements and neighboring information exchange over a communication graph, a distributed estimator is designed so that each local estimator can cooperatively reconstruct the global operating state. Furthermore, the estimator gain is designed, and convergence analysis is provided

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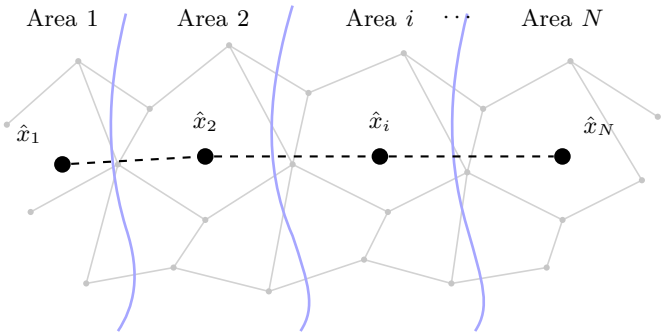

Fig. 1: Partition of the active distribution grid into multiple areas. The faded background represents the physical distribution network, the light boundaries indicate regional partitions, and the dashed links denote the communication topology among local estimators for distributed state estimation.

to establish the effectiveness of the proposed estimation scheme.

The remainder of this paper is organized as follows. Section II formulates the distributed state estimation problem for ADGs with integrated BESSs. Section III presents the distributed estimator and gain design. Section IV provides the convergence analysis. Section V reports the case studies. Finally, Section VI concludes the paper.

II. Problem Formulation

Consider an active distribution grid (ADG) with integrated battery energy storage systems (BESSs), which is partitioned into N interconnected areas indexed by

$$\mathcal{V} = \{1, 2, \dots, N\}.$$

Each area is associated with one local estimator. As illustrated in Fig. 1, the active distribution grid is partitioned into multiple areas according to geographical proximity or electrical coupling. Each black dot represents the estimate generated by one local estimator, while the dashed links indicate the communication topology used for distributed information exchange.

To describe the information exchange among local estimators, a communication graph is introduced as

$$\mathcal{G}_c = (\mathcal{V}, \mathcal{E}_c, \mathcal{A}_c),$$

where $\mathcal{E}_c \subseteq \mathcal{V} \times \mathcal{V}$ denotes the set of communication links and $\mathcal{A}_c = [a_{ij}^c] \in \mathbb{R}^{N\times N}$ is the corresponding weighted adjacency matrix. An edge $(j, i) \in \mathcal{E}_c$ indicates that estimator i can receive information from estimator j, and $a_{ij}^c > 0$ is the associated communication weight; otherwise, $a_{ij}^c = 0$.

A. Area-Level State Model

To describe the operating condition of the active distribution grid (ADG) with integrated battery energy storage systems (BESSs), the area-level state is constructed by combining the electrical-network states and the storage-related states.

First, for area i, the electrical-state vector is defined as

$$x_{i,e}(k) = \mathrm{col}\big(V_{i,1}(k), \theta_{i,1}(k), \dots, V_{i,n_i^v}(k), \theta_{i,n_i^v}(k)\big),$$

where $V_{i,\ell}(k)$ and $\theta_{i,\ell}(k)$ denote the voltage magnitude and phase angle of the ℓth selected bus in area i, respectively, and n_i^v is the number of selected buses.

Second, to capture the influence of the integrated BESSs on the network operation, storage-related variables can also be included in the state description whenever a more accurate dynamic representation is needed. In particular, the storage-state vector of area i is defined as

$$x_{i,s}(k) = \mathrm{col}\big(SOC_{i,1}(k), SOC_{i,2}(k), \dots, SOC_{i,n_i^b}(k)\big),$$

where $SOC_{i,b}(k)$ denotes the state of charge (SOC) of the bth BESS in area i, and n_i^b is the number of BESS units in that area.

For each BESS unit, the SOC dynamics are described by

$$\begin{aligned} SOC_{i,b}(k+1) =& SOC_{i,b}(k) - \frac{\eta_{i,b}^d P_{i,b}^d(k)\Delta t}{E_{i,b}^{\max}} \\ &+ \frac{\eta_{i,b}^c P_{i,b}^c(k)\Delta t}{E_{i,b}^{\max}} + \omega_{i,b}^{soc}(k), \end{aligned} \tag{1}$$

where $P_{i,b}^d(k)$ and $P_{i,b}^c(k)$ are the discharging and charging powers, respectively, $\eta_{i,b}^d$ and $\eta_{i,b}^c$ are the corresponding efficiencies, $E_{i,b}^{\max}$ is the rated energy capacity, and $\omega_{i,b}^{soc}(k)$ denotes the modeling uncertainty and disturbance.

It should be emphasized that the electrical states and the storage states are physically coupled rather than independent. On the one hand, the charging/discharging behavior of the BESSs changes the local active power injection and hence affects the voltage profile, branch power flow, and power balance. On the other hand, the network operating condition constrains the storage power exchange and influences the SOC evolution. Therefore, the electrical-network states and the storage states should be described in a unified area-level model.

Accordingly, the overall state of area i is defined as

$$x_i(k) = \begin{bmatrix} x_{i,e}(k) \\ x_{i,s}(k) \end{bmatrix} \in \mathbb{R}^{n_i},$$

where n_i is the dimension of the aggregated area-level state.

Moreover, due to the electrical interconnection among different areas, the state evolution of area i is influenced not only by its own local states but also by the states of other areas through physical coupling. Hence, the area-level dynamics are described by the following coupled discrete-time model:

$$x_i(k+1) = A_{ii}x_i(k) + \sum_{j=1,\, j\neq i}^{N} A_{ij}x_j(k) + w_i(k), \tag{2}$$

where A_{ii} denotes the local state-transition matrix of area i, A_{ij} characterizes the physical coupling effect from area

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j to area i for $j \neq i$, and $w_i(k)$ is the bounded process disturbance and modeling uncertainty.

B. Measurement Model

Each local estimator has access to local measurements collected by intelligent electronic devices and storage monitoring units. The measurement model of area i is described by

$$y_i(k) = h_i\big(x_i(k)\big) + v_i(k), \tag{3}$$

where $y_i(k) \in \mathbb{R}^{p_i}$ is the local measurement vector, $h_i(\cdot)$ is a nonlinear measurement function, and $v_i(k)$ denotes the measurement noise.

The available measurements may include bus voltage magnitudes, active/reactive power injections, branch power flows, and battery-storage-related measurements. Around an operating point, (3) can be linearized as

$$y_i(k) = c_i(k)x_i(k) + v_i(k), \tag{4}$$

where $c_i(k)$ is the local measurement matrix.

C. Inter-Area Information Exchange

To improve the estimation accuracy and consistency among areas, local estimators exchange their state information through the communication graph $\mathcal{G}_c$. The information received by estimator i from estimator j is represented by

$$z_{ij}(k) = \hat{x}_j(k), \qquad j \in \mathcal{N}_i^c, \tag{5}$$

where $\mathcal{N}_i^c$ denotes the communication neighbor set of area i, and $\hat{x}_j(k)$ is the state estimate generated by estimator j.

It is worth noting that the communication neighbor set $\mathcal{N}_i^c$ is introduced for distributed information exchange, while the physical coupling among area states is captured by the coupling terms in (2). Therefore, the communication topology used by the distributed estimator does not need to coincide with the physical coupling pattern of the underlying power network.

D. Problem Statement

Let $\hat{x}_i(k)$ denote the estimate of $x_i(k)$ generated by the local estimator in area i, and define the estimation error as

$$e_i(k) = x_i(k) - \hat{x}_i(k).$$

The problem considered in this paper can be stated as follows.

Problem 1: Given the coupled area-level state model (2), the measurement model (4), and the communication-based neighboring information exchange mechanism (5), design a distributed state estimation scheme for each area $i \in \mathcal{V}$ such that the regional operating states of the active distribution grid with integrated battery energy storage can be accurately reconstructed in a cooperative manner.

III. Distributed State Estimator

A. Estimator Design

Based on the coupled dynamics of the active distribution grid, all regional states are first stacked into a global state vector

$$x(k) = \mathrm{col}\big(x_1(k), x_2(k), \ldots, x_N(k)\big) \in \mathbb{R}^n,$$

where

$$n = \sum_{i=1}^{N} n_i.$$

Accordingly, the overall ADG can be described by the following coupled large-scale system:

$$x(k+1) = Ax(k) + w(k), \tag{6}$$

where $A \in \mathbb{R}^{n\times n}$ is the global state-transition matrix obtained by assembling the local dynamics and inter-area coupling terms, and $w(k) \in \mathbb{R}^n$ denotes the stacked process disturbance.

For each estimator $i \in \mathcal{V}$, the local measurement model (4) is written as

$$y_i(k) = C_i(k)x(k) + v_i(k), \tag{7}$$

where $y_i(k) \in \mathbb{R}^{p_i}$ is the local measurement vector, $C_i(k) \in \mathbb{R}^{p_i\times n}$ is the local measurement matrix with respect to the global state, and $v_i(k) \in \mathbb{R}^{p_i}$ is the measurement noise.

Since each estimator only has access to local measurements, a single estimator generally cannot reconstruct the global coupled state accurately on its own. Therefore, a distributed cooperative estimation scheme is introduced by combining local measurement correction and neighboring information fusion over the communication graph $\mathcal{G}_c$.

Let $\hat{x}_i(k) \in \mathbb{R}^n$ denote the estimate of the global state $x(k)$ generated by estimator i. Then, the distributed state estimator is designed as

$$\begin{aligned}\hat{x}_i(k+1) =& A\hat{x}_i(k) + L_i\big(y_i(k) - C_i(k)\hat{x}_i(k)\big) \\ &+ \sum_{j\in\mathcal{N}_i^c} K_{ij}\big(\hat{x}_j(k) - \hat{x}_i(k)\big),\end{aligned} \tag{8}$$

where $L_i \in \mathbb{R}^{n\times p_i}$ is the local measurement gain matrix, $K_{ij} \in \mathbb{R}^{n\times n}$ is the neighboring fusion gain associated with the communication link from estimator j to estimator i, and $\mathcal{N}_i^c$ denotes the communication neighbor set of estimator i.

The estimator in (8) consists of three parts. The first term $A\hat{x}_i(k)$ is the model-based prediction of the global coupled state. The second term $L_i\big(y_i(k) - C_i(k)\hat{x}_i(k)\big)$ is the local innovation correction term, which updates the estimate using the local measurement residual. The third term $\sum_{j\in\mathcal{N}_i^c} K_{ij}\big(\hat{x}_j(k) - \hat{x}_i(k)\big)$ is the neighboring information fusion term, which incorporates the global state estimates received from communication neighbors and drives the local estimators toward consensus.

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For notational convenience, define the local estimation error of estimator i as

$$e_i(k) = x(k) - \hat{x}_i(k). \tag{9}$$

By combining (6) and (8), the estimation error dynamics of estimator i can be obtained as

$$\begin{aligned} e_i(k+1) = &\big(A - L_i C_i(k)\big)e_i(k) \\ &- \sum_{j\in\mathcal{N}_i^c} K_{ij}\big(e_i(k) - e_j(k)\big) + w(k) - L_i v_i(k), \end{aligned} \tag{10}$$

where $\tilde{w}(k) = w(k) - L_i v_i(k)$ represents the combined effect of process disturbance and measurement noise on the estimation error dynamics.

In this following, we propose a tailored design methodology for the local measurement gain L_i and the neighboring fusion gain K_{ij}. The fundamental strategy is to decouple the error dynamics into observable and unobservable subspaces for each local estimator, ensuring that L_i handles the identifiable modes while K_{ij} stabilizes the remaining dynamics through consensus. We assume that the operating point of the active distribution grid remains relatively stable over a specific time horizon, such that the linearized measurement matrix $C_i(k)$ can be treated as a constant matrix, denoted as C_i. This assumption allows for a time-invariant decomposition of the state space into observable and unobservable subspaces.

For each area $i \in \mathcal{V}$, let $n_{i,o}$ denote the dimension of the observable subspace of the pair $(A, C_i(k))$. There exists a non-singular transformation matrix $T_i \in \mathbb{R}^{n\times n}$ such that the global state-transition matrix A and the local measurement matrix C_i are transformed into the following observer canonical form:

$$\tilde{A}_i = T_i A T_i^{-1} = \begin{bmatrix} A_{i,o} & 0 \\ A_{i,uo} & A_{i,\bar{o}} \end{bmatrix}, \quad \tilde{C}_i = C_i T_i^{-1} = \begin{bmatrix} C_{i,o} & 0 \end{bmatrix} \tag{11}$$

where the pair $(A_{i,o}, C_{i,o})$ is completely observable. Accordingly, the estimation error in the transformed coordinates is partitioned as $\tilde{e}_i(k) = T_i e_i(k) = \mathrm{col}(\tilde{e}_{i,o}(k), \tilde{e}_{i,\bar{o}}(k))$.

The local gain L_i is designed to prioritize the correction of the observable components $\tilde{e}_{i,o}$. We define the structure of L_i as:

$$L_i = T_i^{-1} \begin{bmatrix} L_{i,o} \\ 0 \end{bmatrix} \tag{12}$$

where $L_{i,o} \in \mathbb{R}^{n_{i,o}\times p_i}$ is the local observer gain. By selecting $L_{i,o}$ such that the matrix $(A_{i,o} - L_{i,o}C_{i,o})$ is Schur stable (i.e., its eigenvalues lie within the unit circle), the local innovation term effectively suppresses the errors in the observable subspace $\mathcal{O}_i$.

For the unobservable subspace $\bar{\mathcal{O}}_i$, where local measurements provide no information, we rely on the consensus-based fusion mechanism. The fusion gain K_{ij} is structured to dominate the stabilization of these unobservable modes:

$$K_{ij} = \gamma_i a_{ij}^c \cdot T_i^{-1} \begin{bmatrix} 0 & 0 \\ 0 & I_{n-n_{i,o}} \end{bmatrix} T_i \tag{13}$$

where $\gamma_i > 0$ is the consensus scalar. This design ensures that the error components in $\bar{\mathcal{O}}_i$ are driven toward the state estimates of neighboring areas $j \in \mathcal{N}_i^c$. Under the conditions of global observability and communication network connectivity, information propagates from areas where specific modes are observable to areas where they are unobservable.

B. Stability Analysis

Substituting the structured gains into the error dynamics (10), and considering the noise-free case for clarity, the transformed error dynamics for area i are given by:

$$\begin{aligned} \tilde{e}_i(k+1) = &\begin{bmatrix} A_{i,o} - L_{i,o}C_{i,o} & 0 \\ A_{i,uo} & A_{i,\bar{o}} \end{bmatrix} \tilde{e}_i(k) \\ &- \gamma_i \sum_{j\in\mathcal{N}_i^c} a_{ij}^c \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} \big(\tilde{e}_i(k) - T_i T_j^{-1} \tilde{e}_j(k)\big) \end{aligned} \tag{14}$$

Theorem 1: Consider the disturbance-free system (14) with the structured gains L_i and K_{ij} as defined above. The estimation error $e_i(k) = \hat{x}_i(k) - x(k)$ is asymptotically stable if:

1) For all $i \in \mathcal{V}$, the local gain $L_{i,o}$ satisfies $\rho(A_{i,o} - L_{i,o}C_{i,o}) < 1$.
2) The global system is collectively observable, i.e., the pair $(A, \mathrm{col}(C_1, \ldots, C_N))$ is observable for any index k.
3) The communication graph $\mathcal{G}_c$ is strongly connected.

Proof: To prove asymptotic stability, we examine the evolution of the error components in the two subspaces defined by the transformation T_i.

First, consider the observable subspace $\tilde{e}_{i,o}(k)$. From (14), notice that the consensus term is structured with a block of zeros in the top row, meaning it does not directly affect the observable subspace dynamics. Thus:

$$\tilde{e}_{i,o}(k+1) = (A_{i,o} - L_{i,o}C_{i,o})\tilde{e}_{i,o}(k) \tag{15}$$

Given that $\rho(A_{i,o} - L_{i,o}C_{i,o}) < 1$, it follows that $\lim_{k\to\infty} \tilde{e}_{i,o}(k) = 0$ for all $i \in \mathcal{V}$.

Second, we analyze the unobservable subspace $\tilde{e}_{i,\bar{o}}(k)$. The dynamics can be written as:

$$\begin{aligned} \tilde{e}_{i,\bar{o}}(k+1) = &A_{i,uo}\tilde{e}_{i,o}(k) + A_{i,\bar{o}}\tilde{e}_{i,\bar{o}}(k) \\ &- \gamma_i \sum_{j\in\mathcal{N}_i^c} a_{ij}^c (\tilde{e}_{i,\bar{o}}(k) - [\tilde{e}_j(k)]_{trans}) \end{aligned} \tag{16}$$

where $[\tilde{e}_j(k)]_{trans}$ represents the transformed influence of neighbor j. Since $\tilde{e}_{i,o}(k) \to 0$, the term $A_{i,uo}\tilde{e}_{i,o}(k)$ acts as a vanishing disturbance. The stability of the remaining term depends on the consensus protocol.

Define a global Lyapunov function $V(k) = \sum_{i=1}^N \tilde{e}_{i,\bar{o}}^T(k) P_i \tilde{e}_{i,\bar{o}}(k)$. Under the condition of global observability, any mode that is unobservable in Area i must be observable in at least one other Area $m \in \mathcal{V}$. Because $\mathcal{G}_c$ is strongly connected, the stabilizing information from Area m (where the local L_m suppresses

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the error) propagates to Area i via the consensus term. By choosing a sufficiently small γ_i, the coupling term ensures that the disagreement between local estimates vanishes, driving the collective unobservable error to zero. Consequently, $\lim_{k\to\infty} e_i(k) = 0$ for all i. This ends the proof. ■

We back to (10). We assume that the combined disturbance is bounded, i.e., there exists a constant $\bar{w} > 0$ such that $\|\tilde{w}_i(k)\| \leq \bar{w}$ for all $i \in \mathcal{V}$ and $k \geq 0$. Under the transformation T_i defined in the previous section, the perturbed error dynamics in the transformed coordinates become:

$$\begin{aligned}\tilde{e}_i(k+1) = &\begin{bmatrix} A_{i,o} - L_{i,o}C_{i,o} & 0 \\ A_{i,uo} & A_{i,\bar{o}} \end{bmatrix} \tilde{e}_i(k) \\ &- \gamma_i \sum_{j\in\mathcal{N}_i^c} a_{ij}^c \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} \left(\tilde{e}_i(k) - T_i T_j^{-1} \tilde{e}_j(k)\right) \\ &+ T_i \tilde{w}_i(k)\end{aligned} \tag{17}$$

Theorem 2: Consider the distributed error system (17) under the structured gain design. If the conditions of Theorem 3 are satisfied and the combined disturbance $\tilde{w}_i(k)$ is bounded, then the estimation error $e_i(k)$ is uniformly ultimately bounded (UUB).

Proof: The proof utilizes the properties of cascaded Input-to-State Stable (ISS) systems.

First, we examine the observable subspace $\tilde{e}_{i,o}(k)$. Let $\tilde{\eta}_{i,o}(k)$ denote the upper block of the transformed noise $T_i\tilde{w}_i(k)$. The dynamics are given by:

$$\tilde{e}_{i,o}(k+1) = (A_{i,o} - L_{i,o}C_{i,o})\tilde{e}_{i,o}(k) + \tilde{\eta}_{i,o}(k) \tag{18}$$

Since $\rho(A_{i,o} - L_{i,o}C_{i,o}) < 1$, the unforced system is exponentially stable. According to ISS theory for discrete-time linear systems, the bounded input $\tilde{\eta}_{i,o}(k)$ ensures that $\tilde{e}_{i,o}(k)$ converges to a bounded set $\Omega_{i,o}$, whose radius is proportional to $\sup_k \|\tilde{\eta}_{i,o}(k)\|$.

Second, consider the unobservable subspace $\tilde{e}_{i,\bar{o}}(k)$. Its dynamics are:

$$\begin{aligned}\tilde{e}_{i,\bar{o}}(k+1) =& A_{i,\bar{o}}\tilde{e}_{i,\bar{o}}(k) - \gamma_i \sum_{j\in\mathcal{N}_i^c} a_{ij}^c (\tilde{e}_{i,\bar{o}}(k) - [\tilde{e}_j(k)]_{trans}) \\ &+ A_{i,uo}\tilde{e}_{i,o}(k) + \tilde{\eta}_{i,\bar{o}}(k)\end{aligned} \tag{19}$$

where $\tilde{\eta}_{i,\bar{o}}(k)$ is the lower block of $T_i\tilde{w}_i(k)$. This subsystem can be viewed as a consensus-based error system driven by two bounded terms: the observable error $\tilde{e}_{i,o}(k)$ and the local noise $\tilde{\eta}_{i,\bar{o}}(k)$.

Under the assumption of strong connectivity of $\mathcal{G}_c$ and global observability, the collective consensus dynamics are stable. Since the cascade of two ISS systems is also ISS, the global error vector $e(k) = \text{col}(e_1, \ldots, e_N)$ remains bounded. Specifically, there exist constants $c > 0$ and $0 < \lambda < 1$ such that:

$$\|e(k)\| \leq c\lambda^k \|e(0)\| + G(\bar{w}) \tag{20}$$

where $G(\bar{w})$ is a non-decreasing function of the noise bound. Thus, the estimation error is UUB, which ends the proof. ■

IV. Simulation

To evaluate the performance of the proposed distributed state estimation method, numerical simulations are conducted on a modified IEEE 33-bus active distribution grid (ADG) with integrated battery energy storage systems (BESSs). The physical system is based on the IEEE 33-bus radial distribution network with a nominal voltage level of 12.66 kV. To represent an active distribution grid, four BESS units are integrated at buses 6, 14, 22, and 30, respectively. The global state vector is defined as

$$x(k) = \text{col}\left(V_1, \theta_1, \ldots, V_{33}, \theta_{33}, SOC_1, \ldots, SOC_4\right), \tag{21}$$

where the overall state dimension is $n = 33 \times 2 + 4 = 70$. The coupled state-transition matrix is obtained from a linearized model around the nominal operating point, where the BESS SOC dynamics are incorporated into the global state-space representation.

To enable distributed estimation, the ADG is partitioned into $N = 4$ interconnected areas according to electrical proximity. Area 1 contains buses 1–12 and BESS 1, Area 2 contains buses 13–18 and BESS 2, Area 3 contains buses 19–25 and BESS 3, and Area 4 contains buses 26–33 and BESS 4. The corresponding modified IEEE 33-bus system and the area partition are illustrated in Fig. 2. The communication graph $\mathcal{G}_c$ is selected as a strongly connected ring topology, where each local estimator exchanges its global state estimate with its adjacent neighbors. The communication weights are set as $a_{ij}^c = 1$ for all active communication links.

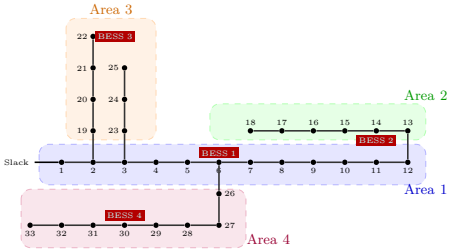

Fig. 2: Modified IEEE 33-bus system partitioned into four local estimation areas with integrated BESS units.

For each area, the local measurement vector consists of local voltage magnitudes, local phase-angle-related measurements, and local SOC measurements. Therefore, although each estimator only has access to regional measurements, it is required to reconstruct the full global state by combining local innovation correction and neighboring information exchange. The simulation horizon is chosen as $T = 120$ sampling steps. To emulate non-ideal operating conditions, bounded process disturbances and measurement noises are introduced. The process disturbance represents load fluctuations and modeling mismatch, while

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the measurement noise reflects the limited precision of local intelligent electronic devices and BESS monitoring units. The local innovation gains are designed according to the local measurement models, and the neighboring fusion gains are selected in the form

$$K_{ij} = a_{ij}^{c}\gamma I_n, \qquad j \in \mathcal{N}_i^{c}, \tag{22}$$

where γ is the scalar consensus gain.

The estimation performance is evaluated by the overall root-mean-square error (RMSE), the voltage RMSE, the SOC RMSE, and the consensus error among local estimators. The overall RMSE is defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{NT}\sum_{i=1}^{N}\sum_{k=1}^{T}\|x(k) - \hat{x}_i(k)\|^2}, \tag{23}$$

the consensus error is defined as

$$\mathrm{CE}(k) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\|\hat{x}_i(k) - \bar{x}(k)\|^2}, \tag{24}$$

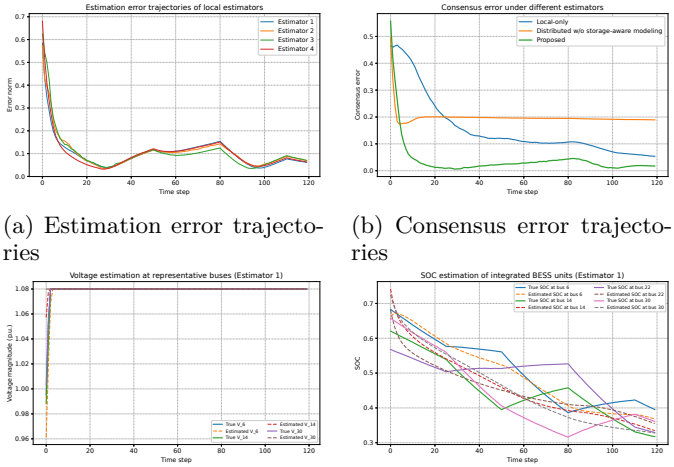
where

$$\bar{x}(k) = \frac{1}{N}\sum_{i=1}^{N}\hat{x}_i(k),$$

and the voltage and SOC estimation errors are evaluated by the corresponding RMSE values over the simulation horizon.

The simulation results show that the proposed estimator can effectively reconstruct the global coupled state from mismatched initial conditions. In particular, the overall RMSE, voltage RMSE, and SOC RMSE obtained in the simulation are 0.131496, 0.012254, and 0.039850, respectively, while the final consensus error is 0.017449. These results indicate that the proposed distributed estimation scheme can achieve accurate global-state reconstruction and good agreement among local estimators.

Figs. 3a–3d further illustrate the effectiveness of the proposed method. Fig. 3a shows that the estimation errors of all local estimators decrease gradually over time, which verifies the convergence of the distributed estimation process. Fig. 3b shows that the consensus error also decreases continuously, indicating that the communication-based neighboring information fusion effectively improves the agreement among local estimators. In addition, Figs. 3c and 3d demonstrate that the estimated voltage and SOC trajectories closely follow the corresponding true trajectories at representative buses and BESS units. These results confirm that the proposed distributed state estimation method can accurately reconstruct both the electrical states and the storage-related states of the active distribution grid.



(a) Estimation error trajectories

(b) Consensus error trajectories

(c) True and estimated voltage magnitudes

(d) True and estimated SOC trajectories

Fig. 3: Overall estimation results under the distributed state estimation method.

V. Conclusion

This study has developed a subspace-based distributed state estimation methodology for active distribution grids integrated with BESS. The research has centered on a systematic approach that has decomposed the global heterogeneous state space into distinct observable and unobservable subspaces for each regional estimator. By leveraging this subspace-driven strategy local gains L_i have been designed to suppress regional errors, while consensus gains K_{ij} have been implemented to synchronize global states via neighborhood information exchange. The analytical framework has established the uniform ultimate boundedness of estimation errors under process and measurement disturbances. Validation on a modified IEEE 33-bus system has confirmed the rapid convergence of all nodal voltages, phase angles, and BESS SOCs, providing a robust and theoretically grounded solution for the real-time supervision of modern power networks.

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