Leader-follower Consensus of Upper-triangular Nonlinear Multi-agent Systems

Chenghui Zhang Le Chang Xianfu Zhang

***Abstract*—This paper is concerned with the leader-follower consensus problem by using both state and output feedback for a class of nonlinear multi-agent systems. The agents considered here are all identical upper-triangular nonlinear systems which satisfy the Lipschitz growth condition. First, it is shown that the leader-follower consensus problem is equivalent to the control design problem of a high-dimensional multi-variable system. Second, by introducing an appropriate state transformation, the control design problem can be converted into the problem of finding a constant parameter, which can be obtained by solving the Lyapunov equation and estimating the nonlinear terms of the given system. At last, an example is given to verify effectiveness of the proposed consensus algorithms.**

***Index Terms*—Multi-agent systems, leader-follower consensus, upper-triangular nonlinear systems, Lipschitz condition.**

I. INTRODUCTION

IN recent years, there has been an increasing research interest in the coordinated control problems of the multi-agent systems[1−11], which have wide applications in many fields, such as biology, robotics, communications and sensor networks, etc.

There are many unsolved problems in the research area of multi-agent systems. One difficult problem is that the structures of many multi-agent systems, especially some nonlinear multi-agent systems, are too complex to deal with. A lot of literature has studied this problem, for example, [2, 3, 10] studied the linear systems, [5, 8, 9, 11] dealt with the second-order agent systems, [4] presented necessary and sufficient conditions for the consensus of multi-agent systems described by Vicsek′s model. Another difficult problem is that the multi-agent systems greatly differ from each other in the communication topologies. Reference [1] studied the problem that the information received by each agent is corrupted by measurement noises. References [11, 12] addressed consensus problems when the information is with or without communication delay. Reference [13] worked on the consensus behavior of multi-agent systems under digital network topology. The consensus problem with input constraints was considered in [14].

A critical problem for coordinated control is the consensus problem, which is to design appropriate protocols and algorithms such that the group of agents can reach a consensus on the shared information in the presence of limited and unreliable information exchange. Recently, the consensus problem has been studied for multi-agent systems in a general form. Reference [2] studied the consensus problem for multi-agent linear dynamic systems. Reference [6] designed the global H_∞ consensus algorithms for a class of multi-agent nonlinear systems with Lipschitz non-linearity and directed communication graphs.

Due to the existence of abundant nonlinear physical systems in practical and the need of precise simulation to describe many commonly observed phenomena, the study of nonlinear multi-agent systems is getting more and more important. Because of complexity of nonlinearity, there is no control procedure which can be applied to all nonlinear multi-agent systems. But there are several effective ways to design controllers for some nonlinear systems that can be transformed into specific nonlinear forms. The lower-triangular nonlinear multi-agent systems have been studied in [7, 15, 16], but to our best knowledge, there is no literature which has handled the upper-triangular nonlinear multi-agent systems.

In our work, the leader-follower consensus problem is studied for the upper-triangular nonlinear multi-agent systems with fixed (time-invariant) communication topologies. The consensus protocols of both the local state and observer-based dynamic output are considered. Compared to the lower-triangular systems studied in [7, 15, 16], the consensus problem of the upper-triangular nonlinear systems is harder, as the input appears in every nonlinear term.

Inspired by [17−20], we introduce a rescaling transformation and have a new design freedom to stabilize the errors between the leader signals and follower signals. When the full state is not available, observers can be designed such that their signals asymptotically approach the signals of agents. As a result, using the observer-based compensation, dynamic output consensus protocol can be constructed. An example is given to illustrate the proposed consensus algorithms at the end of this paper.

Throughout this work, $\|\cdot\|$ denotes the Euclidean norm for a vector, or the induced Euclidean norm for a matrix.

II. GRAPH THEORY AND PROBLEM STATEMENT

A. Graph Theory

In this section, we present some definitions, notations and lemmas in graph theory, which will be used in our paper. A simple graph is an undirected graph if it has no self-loops and no more than one edge between any two different nodes. The simple graph denoted by $G(\mathcal{V}, \Im, \mathcal{A})$ consists of an N node set $\mathcal{V}$, an edge set $\Im$ and a weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbf{R}^{N\times N}$. We denote the nodes in $\mathcal{V}$ as $\{s_1, s_2, \cdots, s_N\}$.

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The edge of the graph G, $e_{ij} = (s_i, s_j)$, belongs to $\Im$. The graph is undirected, i.e., once $e_{ij} \in \Im$, then $e_{ji} \in \Im$. As the graph is a simple graph, the adjacency matrix is defined as $a_{ii} = 0$ and $a_{ij} = a_{ji} \geq 0\,(i \neq j)$, where $a_{ij} > 0$ if and only if $e_{ij} \in \Im$. A path in G between s_1 and s_j is a sequence of edges of the form $(s_1, s_k), k = 1, 2, \cdots, j$. The graph G is said to be connected if there exists a path between any two nodes of G. The neighbor set of node s_i is defined as $N_i = \{s_j \in \mathcal{V} : e_{ij} \in \Im\}$. The degree of G is a diagonal matrix $\mathcal{D} = \mathrm{diag}\{d_1, \cdots, d_N\}$, where $d_i = \sum_{s_j \in N_i} a_{ij}$ for $i = 1, 2, \cdots, N$. The Laplacian matrix of graph G is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$. It is apparent that the Laplacian matrix $\mathcal{L}$ is symmetric. H is a subgraph of G, i.e., any two nodes of H are adjacent in H only if they are adjacent in G. Subgraph H of G is called component of G if it is a maximal connected subgraph.

We consider N modes in $\mathcal{V}$ as N agents, whose relationship can be described by the simple and undirected graph G. (s_i, s_j) is an edge of G if and only if agent i and j are neighbors. Moreover, we have another graph $\tilde{G}$ whose node set is $\{s_0\} \cup \mathcal{V}$. G is the subgraph of $\tilde{G}$, and s_0 is considered as the leader. The edge $e_{0j} = (s_0, s_j)$ exists if and only if agent j connects to the leader $(j = 1, 2, \cdots, N)$. The degree matrix of $\tilde{G}$ is denoted by $\mathcal{B} = \mathrm{diag}\{b_1, \cdots, b_N\}$, where $b_i \geq 0$ is the adjacency weight between agent i and the leader. $b_i = 0$ means that agent i does not connect to the leader. $\tilde{G}$ is connected if at least one agent in each component of G is connected with the leader. We define $\hat{\mathcal{L}} = \mathcal{L} + \mathcal{B}$, and have the following useful lemma about $\hat{\mathcal{L}}$.

Lemma 1[5, 7]**.** If graph $\tilde{G}$ is connected, then the symmetric matrix $\hat{\mathcal{L}}$ associated with $\tilde{G}$ is positive definite.

B. Problem Statement

In this paper, we deal with a group of $N + 1$ agents with identical upper-triangular nonlinear dynamics, where the agent indexed by 0 is referred as the leader and the other agents indexed by $1, 2, \cdots, N$ are called followers. The communication topology of the $N + 1$ agents is denoted by $\tilde{G}$, while the topology of the N followers is denoted by G. For the agents, we have the following assumptions.

Assumption 1. All follower agents know the input of the leader, and the information between the leader and any follower agent is unidirectional, i.e., the leader receives no information from any follower agent.

Assumption 2. The communication topology of the $N + 1$ agents is connected, i.e., the graph $\tilde{G}$ is connected.

For agent k $(k = 0, 1, \cdots, N)$, the system has the following form

$$\begin{cases} \dot{x}_{k,1} = x_{k,2} + f_1(t, x_{k,3}, x_{k,4}, \cdots, x_{k,n}, u_k), \\ \dot{x}_{k,2} = x_{k,3} + f_2(t, x_{k,4}, \cdots, x_{k,n}, u_k), \\ \qquad\qquad \vdots \\ \dot{x}_{k,n-2} = x_{k,n-1} + f_{n-2}(t, x_{k,n}, u_k), \\ \dot{x}_{k,n-1} = x_{k,n} + f_{n-1}(t, u_k), \\ \dot{x}_{k,n} = u_k, \\ y_k = x_{k,1}, \end{cases} \tag{1}$$

where $x_{k,i} \in \mathbf{R}\,(i = 1, 2, \cdots, n)$, $y_k \in \mathbf{R}$ and $u_k \in \mathbf{R}$ represent the state, output, and input of the kth agent, respectively, and nonlinear function $f_i\,(i = 1, 2, \cdots, n-1)$ represents the nonlinear effect within the kth agent, and satisfy the following growth assumption.

Assumption 3. For $p, q = 0, 1, \cdots, N$, and any $(t, x_{p,i+2}, \cdots, x_{p,n}, u_p)$, $(t, x_{q,i+2}, \cdots, x_{q,n}, u_q) \in \mathbf{R}^{n-i+1}$ $(i = 1, 2, \cdots, n-1)$, there exists a constant $c > 0$, such that

$$|f_i(t, x_{p,i+2}, \cdots, x_{p,n}, u_p) - f_i(t, x_{q,i+2}, \cdots, x_{q,n}, u_q)| \leq c\left(\sum_{j=i}^{n-1} |x_{p,j+2} - x_{q,j+2}| + |u_p - u_q|\right), \tag{2}$$

where $x_{p,n+1} = x_{q,n+1} = 0$.

Remark 1. Reference [7] considered the leader-follower consensus problem of a class of multi-agent systems in the lower-triangular form. Hence, the problem considered here can be viewed as the counterpart of that in [7]. In our work, the input of the kth agent can be allowed to be included in all nonlinear terms of the kth agent, which makes our design procedure more complicated, compared with [7].

System (1) can be rewritten in the matrix form,

$$\dot{\boldsymbol{x}}_k = A\boldsymbol{x}_k + Bu_k + \boldsymbol{f}(t, \boldsymbol{x}_k, u_k), \tag{3}$$

where $\boldsymbol{x}_k = [x_{k,1}, x_{k,2}, \cdots, x_{k,n}]^{\mathrm{T}}$, $B = [0, 0, \cdots, 0, 1]^{\mathrm{T}}$, $\boldsymbol{f}(t, \boldsymbol{x}_k, u_k) = [f_1(t, x_{k,3}, \cdots, x_{k,n}, u_k), \cdots, f_{n-1}(t, u_k), 0]^{\mathrm{T}}$, and

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

Definition 1[7]**.** The leader-follower consensus problem of the nonlinear multi-agent system (1) can be solved by consensus protocol u_k, if and only if under the protocol u_k, for any initial condition $\boldsymbol{x}_k(0)\,(k = 1, 2, \cdots, N)$, the state $\boldsymbol{x}_k(t)$ of the follower agent k asymptotically approaches the state $\boldsymbol{x}_0(t)$ of the leader, as $t \to +\infty$. That is,

$$\lim_{t \to +\infty} \|\boldsymbol{x}_k(t) - \boldsymbol{x}_0(t)\| = 0, \quad k = 1, 2, \cdots, N.$$

Our objective is to design consensus protocols by both the state and output feedback, which solve the leader-follower consensus problem of the nonlinear multi-agent system (1).

As defined above, $\tilde{G}$ and G are the communication topologies of the $N + 1$ agents and the N follower agents, respectively. We use matrix $\mathcal{L}$ and $\mathcal{B}$ to denote the Laplacian matrix of G and the degree matrix of $\tilde{G}$, respectively. We get $\hat{\mathcal{L}}$ by $\hat{\mathcal{L}} = \mathcal{L} + \mathcal{B}$. According to Lemma 1, $\hat{\mathcal{L}}$ is positive definite. We have the following useful lemma about $\hat{\mathcal{L}}$.

Lemma 2[7]**.** If $\hat{\mathcal{L}}$ is positive definite, then there exists a row vector K such that the matrix $I_N \otimes A + \hat{\mathcal{L}} \otimes BK$ is Hurwitz. Furthermore, there exists a positive definite matrix P satisfying

$$P(I_N \otimes A + \hat{\mathcal{L}} \otimes BK) + (I_N \otimes A + \hat{\mathcal{L}} \otimes BK)^{\mathrm{T}} P = -2I. \tag{4}$$

III. MAIN RESULTS

A. State Feedback Control

Our main result of the state feedback control problem with the full states known, can be summarized as the following theorem.

Theorem 1. Under Assumptions 1∼3, the leader-follower consensus problem of the nonlinear multi-agent system (1) can be solved by the consensus protocol of the following form, for $k=1,2,\cdots,N$,

$$u_k=\frac{1}{L^n}KH\left(\sum_{j=1}^N a_{kj}(\boldsymbol{x}_k-\boldsymbol{x}_j)+b_k(\boldsymbol{x}_k-\boldsymbol{x}_0)\right)+u_0, \tag{5}$$

where K is a row vector given in Lemma 2, and diagonal matrix $H=\mathrm{diag}\{1,L,\cdots,L^{n-1}\}$, with $L\geq 1$ being a constant large enough.

Proof. For the consensus problem, we need to consider the state errors between the leader and followers.

For $k=1,2,\cdots,N$, let $e_{k,i}=x_{k,i}-x_{0,i}\,(i=1,2,\cdots,n)$, we have

$$\begin{cases}\dot e_{k,1}=e_{k,2}+\bar f_{k,1},\\ \dot e_{k,2}=e_{k,3}+\bar f_{k,2},\\ \quad\vdots\\ \dot e_{k,n-1}=e_{k,n}+\bar f_{k,n-1},\\ \dot e_{k,n}=\Delta u_k,\end{cases} \tag{6}$$

where $\Delta u_k=u_k-u_0$, and $\bar f_{k,i}=f_i(t,x_{k,i+2},\cdots,x_{k,n},u_k)-f_i(t,x_{0,i+2},\cdots,x_{0,n},u_0)$, $i=1,2,\cdots,n-1$.

The consensus error dynamics (6) can be rewritten as

$$\dot{\boldsymbol{e}}_k=A\boldsymbol{e}_k+B\Delta u_k+\bar{\boldsymbol{f}}_k, \tag{7}$$

where $\bar{\boldsymbol{f}}_k=[\bar f_{k,1},\cdots,\bar f_{k,n-1},0]^{\mathrm T}$, $\boldsymbol{e}_k=[e_{k,1},\cdots,e_{k,n}]^{\mathrm T}$, and A, B are given in (3).

Introducing a transformation of coordinates

$$\boldsymbol{\varepsilon}_k=H\boldsymbol{e}_k,\quad k=1,2,\cdots,N,$$

where $H=\mathrm{diag}\{1,L,\cdots,L^{n-1}\}$, with $L\geq 1$ being a constant to be determined later, system (7) can be converted into

$$\dot{\boldsymbol{\varepsilon}}_k=\frac{1}{L}A\boldsymbol{\varepsilon}_k+L^{n-1}B\Delta u_k+\boldsymbol{F}_k, \tag{8}$$

where $\boldsymbol{F}_k=H\bar{\boldsymbol{f}}_k$.

Denoting $\boldsymbol{\varepsilon}=[\boldsymbol{\varepsilon}_1^{\mathrm T},\boldsymbol{\varepsilon}_2^{\mathrm T},\cdots,\boldsymbol{\varepsilon}_N^{\mathrm T}]^{\mathrm T}$, and using (5), the consensus error dynamics (8) can be further rewritten in the compact form of

$$\dot{\boldsymbol{\varepsilon}}=\frac{1}{L}I_N\otimes A\boldsymbol{\varepsilon}+\frac{1}{L}\hat{\mathcal L}\otimes BK\boldsymbol{\varepsilon}+\boldsymbol{F}, \tag{9}$$

where $\boldsymbol{F}=[\boldsymbol{F}_1^{\mathrm T},\boldsymbol{F}_2^{\mathrm T},\cdots,\boldsymbol{F}_N^{\mathrm T}]^{\mathrm T}$, and $\hat{\mathcal L}$ is defined by the communication topology.

Let $V=\boldsymbol{\varepsilon}^{\mathrm T}P\boldsymbol{\varepsilon}$, where P is given in Lemma 2. The derivative of V along system (9) is given as

$$\begin{aligned}\dot V|_{(9)}=\ &2\boldsymbol{\varepsilon}^{\mathrm T}P\boldsymbol{F}+\frac{1}{L}\boldsymbol{\varepsilon}^{\mathrm T}\big[(I_N\otimes A+\hat{\mathcal L}\otimes BK)^TP+\\ &P(I_N\otimes A+\hat{\mathcal L}\otimes BK)\big]\boldsymbol{\varepsilon}=-\frac{2}{L}\boldsymbol{\varepsilon}^{\mathrm T}\boldsymbol{\varepsilon}+2\boldsymbol{\varepsilon}^{\mathrm T}P\boldsymbol{F}.\end{aligned} \tag{10}$$

To get the estimation of the term $2\boldsymbol{\varepsilon}^{\mathrm T}P\boldsymbol{F}$, the following estimation of Δu_k and $\bar f_{k,i}$, which are contained in $\boldsymbol{F}$, is needed.

Noticing the definition of Δu_k in (5) and (6), we can get the following derivation

$$\begin{aligned}|\Delta u_k|^2=\ &\left|\frac{1}{L^n}KH\left(\sum_{j=1}^N a_{kj}(\boldsymbol{x}_k-\boldsymbol{x}_j)+b_k(\boldsymbol{x}_k-\boldsymbol{x}_0)\right)\right|^2=\\ &\frac{1}{L^{2n}}\left(\sum_{j=1}^N a_{kj}(\boldsymbol{\varepsilon}_k-\boldsymbol{\varepsilon}_j)+b_k\boldsymbol{\varepsilon}_k\right)^{\mathrm T}K^{\mathrm T}K\times\\ &\left(\sum_{j=1}^N a_{kj}(\boldsymbol{\varepsilon}_k-\boldsymbol{\varepsilon}_j)+b_k\boldsymbol{\varepsilon}_k\right)\leq\\ &\frac{1}{L^{2n}}\|K^{\mathrm T}K\|(A_k\boldsymbol{\varepsilon})^{\mathrm T}(A_k\boldsymbol{\varepsilon})\leq\\ &\frac{1}{L^{2n}}\|K^{\mathrm T}K\|\|A_k^{\mathrm T}A_k\|\|\boldsymbol{\varepsilon}\|^2,\end{aligned} \tag{11}$$

where A_k is defined as $A_k=\alpha_k\otimes I_n$, with α_k being the kth row of matrix $\hat{\mathcal L}$.

Using (2) in Assumption 3 and $L\geq 1$, the estimates of $\bar f_{k,i}\,(i=1,2,\cdots,n-1)$ have the following form

$$\begin{aligned}|L^{i-1}\bar f_{k,i}|\leq cL^{i-1}&\left(\sum_{j=i}^{n-1}|e_{k,j+2}|+|\Delta u_k|\right)\leq\\ &c\left(\sum_{j=i}^{n-1}\left|\frac{\varepsilon_{k,j+2}}{L^{j-i+2}}\right|+\frac{1}{L^2}\sqrt{\|K^{\mathrm T}K\|\|A_k^{\mathrm T}A_k\|}\|\boldsymbol{\varepsilon}\|\right)\leq\\ &\frac{c}{L^2}\left(1+\sqrt{\|K^{\mathrm T}K\|\|A_k^{\mathrm T}A_k\|}\right)\|\boldsymbol{\varepsilon}\|,\end{aligned} \tag{12}$$

where $e_{k,n+1}=\varepsilon_{k,n+1}=0$.

With the help of (12), from the definition of $\boldsymbol{F}$ in (9), we can get

$$\begin{aligned}\|\boldsymbol{F}\|^2=&\sum_{k=1}^N\boldsymbol{F}_k^{\mathrm T}\boldsymbol{F}_k=\sum_{k=1}^N(H\bar{\boldsymbol{f}}_k)^{\mathrm T}H\bar{\boldsymbol{f}}_k=\\ &\sum_{k=1}^N(|\bar f_{k,1}|^2+L^2|\bar f_{k,2}|^2+\cdots+L^{2n-4}|\bar f_{k,n-1}|^2)\leq\\ &\frac{\beta^2}{L^4}\|\boldsymbol{\varepsilon}\|^2,\end{aligned} \tag{13}$$

where β is dependent on the known constants $\|K^{\mathrm T}K\|$, $\|A_k^{\mathrm T}A_k\|$ and c. Actually, if $\beta\geq\max_{1\leq k\leq N}c(1+\sqrt{\|K^{\mathrm T}K\|\|A_k^{\mathrm T}A_k\|})$, (13) holds.

With (13) and (10), we have

$$\dot V|_{(9)}\leq-\frac{2}{L}\|\boldsymbol{\varepsilon}\|^2+2\|P\|\frac{\beta}{L^2}\|\boldsymbol{\varepsilon}\|^2.$$

Choosing $L\geq\max\{\beta\|P\|,1\}+\alpha$, where α is any positive constant, we get

$$\dot V|_{(9)}\leq-\frac{2\alpha}{L^2}\|\boldsymbol{\varepsilon}\|^2.$$

Thus, the state $\boldsymbol{\varepsilon}$ exponentially converges to the origin, i.e., consensus errors $\boldsymbol{e}_k\,(k=1,2,\cdots,N)$ converge to the origin.

So we get the full state consensus protocol (5), which solves the leader-follower consensus problem. □

Remark 2. Both $K^{\mathrm{T}}K$ and $A_k^{\mathrm{T}}A_k$ are positive definite matrices, so their norms can be replaced by the maximum of their eigenvalues.

Remark 3. From the definition $A_k=\alpha_k\otimes I_n$ in (11), one can calculate that

$$A_k^{\mathrm{T}}A_k=\ (\alpha_k\otimes I_n)^{\mathrm{T}}(\alpha_k\otimes I_n)= (\alpha_k^{\mathrm{T}}\alpha_k)\otimes I_n.$$

Since $\det(\lambda I_N-\alpha_k^{\mathrm{T}}\alpha_k)=\lambda^{N-1}|\lambda-\alpha_k\alpha_k^{\mathrm{T}}|$, all eigenvalues of $A_k^{\mathrm{T}}A_k$ equal 0 or $\alpha_k\alpha_k^{\mathrm{T}}$. It is easy to get the maximal eigenvalue of $A_k^{\mathrm{T}}A_k$.

Remark 4. It can be seen that the study of upper-triangular multi-agent systems is more difficult than that of lower-triangular multi-agent systems, as the input is allowed to appear in each nonlinear term of the upper-triangular multi-agent systems. To our best knowledge, there is no literature dealing with the upper-triangular multi-agent systems. Compared with the technologies used to study the lower-triangular multi-agent systems in [7, 15, 16], the algorithm in our paper is novel and easy to design.

B. Output Feedback Control

In this section, the output feedback protocol problem is considered when the full state is not available.

First, we construct the observers for all agents and the inputs for the follower agents.

Following the ideas of [17−20], the observer of every agent has the following form, for $k=0,1,\cdots,N$,

$$\begin{cases}\dot{\hat{x}}_{k,1}=\hat{x}_{k,2}+f_1(t,\hat{x}_{k,3},\cdots,\hat{x}_{k,n},u_k)+\frac{c_1}{L}\left(x_{k,1}-\hat{x}_{k,1}\right),\\ \dot{\hat{x}}_{k,2}=\hat{x}_{k,3}+f_2(t,\hat{x}_{k,4},\cdots,\hat{x}_{k,n},u_k)+\frac{c_2}{L^2}\left(x_{k,1}-\hat{x}_{k,1}\right),\\ \qquad\vdots\\ \dot{\hat{x}}_{k,n-2}=\hat{x}_{k,n-1}+f_{n-2}(t,\hat{x}_{k,n},u_k)+\frac{c_{n-2}}{L^{n-2}}\left(x_{k,1}-\hat{x}_{k,1}\right),\\ \dot{\hat{x}}_{k,n-1}=\hat{x}_{k,n}+f_{n-1}(t,u_k)+\frac{c_{n-1}}{L^{n-1}}\left(x_{k,1}-\hat{x}_{k,1}\right),\\ \dot{\hat{x}}_{k,n}=u_k+\frac{c_n}{L^n}\left(x_{k,1}-\hat{x}_{k,1}\right),\end{cases}\tag{14}$$

where $L\geq 1$ will be determined later, and $c_i\geq 0\,(i=1,2,\cdots,n)$ are coefficients of a Hurwitz polynomial

$$q(s)=s^n+c_1s^{n-1}+\cdots+c_{n-1}s+c_n.$$

The input of agent k has the form of

$$u_k=\frac{1}{L^n}KH\left(\sum_{j=1}^{N}a_{kj}(\hat{\boldsymbol{x}}_k-\hat{\boldsymbol{x}}_j)+b_k(\hat{\boldsymbol{x}}_k-\hat{\boldsymbol{x}}_0)\right)+u_0,\tag{15}$$

where diagonal matrix $H=\mathrm{diag}\{1,L,\cdots,L^{n-1}\}$, $\hat{\boldsymbol{x}}_k=[x_{k,1},x_{k,2},\cdots,x_{k,n}]$ is the state variable of observers, and K is the row vector determined in Lemma 2.

For the kth agent, the observer error is defined as $\tilde{x}_{k,j}=\hat{x}_{k,j}-x_{k,j}\,(j=1,2,\cdots,n)$. From (1) and (14), a simple calculation gives

$$\begin{cases}\dot{\tilde{x}}_{k,1}=\tilde{x}_{k,2}+\bar{f}_{k,1}-\frac{c_1}{L}\tilde{x}_{k,1},\\ \dot{\tilde{x}}_{k,2}=\tilde{x}_{k,3}+\bar{f}_{k,2}-\frac{c_2}{L^2}\tilde{x}_{k,1},\\ \qquad\vdots\\ \dot{\tilde{x}}_{k,n-2}=\tilde{x}_{k,n-1}+\bar{f}_{k,n-2}-\frac{c_{n-2}}{L^{n-2}}\tilde{x}_{k,1},\\ \dot{\tilde{x}}_{k,n-1}=\tilde{x}_{k,n}+\bar{f}_{k,n-1}-\frac{c_{n-1}}{L^{n-1}}\tilde{x}_{k,1},\\ \dot{\tilde{x}}_{k,n}=-\frac{c_n}{L^n}\tilde{x}_{k,1},\end{cases}$$

where $\bar{f}_{k,i}=f_i(t,\hat{x}_{k,i+2},\cdots,\hat{x}_{k,n},u_k)-f_i(t,x_{k,i+2},\cdots,x_{k,n},u_k)$, for $i=1,2,\cdots,n-1$.

We denote $\tilde{\boldsymbol{x}}_k=[\tilde{x}_{k,1},\tilde{x}_{k,2},\cdots,\tilde{x}_{k,n}]^{\mathrm{T}}$. Similar to the idea of [20], we introduce the transformation $\boldsymbol{X}_k=H\tilde{\boldsymbol{x}}_k$, and have that

$$\dot{\boldsymbol{X}}_k=\frac{1}{L}\bar{C}\boldsymbol{X}_k+\bar{\boldsymbol{f}}_k,\tag{16}$$

where

$$\bar{C}=\begin{bmatrix}-c_1&1&0&\cdots&0\\-c_2&0&1&\cdots&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\-c_{n-1}&0&0&\cdots&1\\-c_n&0&0&\cdots&0\end{bmatrix},\bar{\boldsymbol{f}}_k=\begin{bmatrix}\bar{f}_{k,1}\\L\bar{f}_{k,2}\\\vdots\\L^{n-2}\bar{f}_{k,n-1}\\0\end{bmatrix}.$$

Since $q(s)$ is a Hurwitz polynomial, it can be concluded that $\bar{C}$ is a stable matrix. Therefore, there exists a positive definite matrix $\bar{P}$ such that

$$\bar{P}\bar{C}+\bar{C}^{\mathrm{T}}\bar{P}=-I.$$

Let $\bar{V}_k=\boldsymbol{X}_k^{\mathrm{T}}\bar{P}\boldsymbol{X}_k\,(k=0,1,\cdots,N)$ be the Lyapunov functions, and we have

$$\dot{\bar{V}}_k|_{(16)}\leq-\frac{1}{L}\|\boldsymbol{X}_k\|^2+2\|\boldsymbol{X}_k\|\|\bar{P}\|\|\bar{\boldsymbol{f}}_k\|.\tag{17}$$

Now we give the estimation of $\|\bar{\boldsymbol{f}}_k\|$ in (17). Noticing $\bar{\boldsymbol{f}}_k=[\bar{f}_{k,1},\cdots,\bar{f}_{k,n-1},0]^{\mathrm{T}}$, from Assumption 3, one has, for $i=1,2,\cdots,n-1$,

$$|L^{i-1}\bar{f}_{k,i}|\leq cL^{i-1}\sum_{j=i}^{n-1}|\tilde{x}_{k,j+2}|\leq\frac{c}{L^2}\sqrt{n}\|\boldsymbol{X}_k\|.$$

Hence

$$\dot{\bar{V}}_k|_{(16)}\leq-\frac{1}{L}\|\boldsymbol{X}_k\|^2+2\|\boldsymbol{X}_k\|\|\bar{P}\|\|\bar{\boldsymbol{f}}_k\|\leq -\frac{1}{L}\|\boldsymbol{X}_k\|^2+\frac{2nc\|\bar{P}\|}{L^2}\|\boldsymbol{X}_k\|^2.\tag{18}$$

Second, we will consider the errors between the observers. For $k=1,2,\cdots,N$, denoting $\boldsymbol{e}_k=\hat{\boldsymbol{x}}_k-\hat{\boldsymbol{x}}_0$, where $\hat{\boldsymbol{x}}_k=[\hat{x}_{k,1},\cdots,\hat{x}_{k,n}]^{\mathrm{T}}$, $\hat{\boldsymbol{x}}_0=[\hat{x}_{0,1},\cdots,\hat{x}_{0,n}]^{\mathrm{T}}$ and introducing a rescaling transformation $\boldsymbol{\varepsilon}_k=H\boldsymbol{e}_k$, with (14), one can obtain

$$\dot{\boldsymbol{\varepsilon}}_k=\frac{1}{L}A\boldsymbol{\varepsilon}_k+L^{n-1}B\Delta u_k+\boldsymbol{F}_k+\frac{1}{L}C\tilde{\boldsymbol{x}}_k-\frac{1}{L}C\tilde{\boldsymbol{x}}_0\tag{19}$$

where A, B are given in (3), $\Delta u_k=u_k-u_0$, $\boldsymbol{F}_k=H\hat{\boldsymbol{f}}_k$, with $\hat{\boldsymbol{f}}_k=\boldsymbol{f}(t,\hat{\boldsymbol{x}}_k,u_k)-\boldsymbol{f}(t,\hat{\boldsymbol{x}}_0,u_0)$, and

$$C=\begin{bmatrix}-c_1&0&\cdots&0\\-c_2&0&\cdots&0\\\vdots&\vdots&\vdots&\vdots\\-c_n&0&\cdots&0\end{bmatrix}.$$

Introducing (15) into (19), we get

$$\begin{aligned}\dot{\boldsymbol{\varepsilon}}_k = &\frac{1}{L}A\boldsymbol{\varepsilon}_k + \frac{1}{L}BK\left(\sum_{j=1}^{N} a_{kj}(\boldsymbol{\varepsilon}_k - \boldsymbol{\varepsilon}_j) + b_k\boldsymbol{\varepsilon}_k\right) + \\ &\boldsymbol{F}_k + \frac{1}{L}C\tilde{\boldsymbol{x}}_k - \frac{1}{L}C\tilde{\boldsymbol{x}}_0, \quad k = 1, 2, \cdots, N. \end{aligned} \tag{20}$$

Denoting $\boldsymbol{\varepsilon} = [\boldsymbol{\varepsilon}_1^{\rm T}, \cdots, \boldsymbol{\varepsilon}_N^{\rm T}]^{\rm T}$, $\boldsymbol{X} = [\tilde{\boldsymbol{x}}_1^{\rm T} - \tilde{\boldsymbol{x}}_0^{\rm T}, \cdots, \tilde{\boldsymbol{x}}_N^{\rm T} - \tilde{\boldsymbol{x}}_0^{\rm T}]^{\rm T}$, $\boldsymbol{F} = [\boldsymbol{F}_1^{\rm T}, \cdots, \boldsymbol{F}_N^{\rm T}]^{\rm T}$, the compact form of (20) can be written as

$$\dot{\boldsymbol{\varepsilon}} = \frac{1}{L}(I_N \otimes A)\boldsymbol{\varepsilon} + \frac{1}{L}(\tilde{\mathcal{L}} \otimes BK)\boldsymbol{\varepsilon} + \frac{1}{L}(I_N \otimes C)\boldsymbol{X} + \boldsymbol{F}, \tag{21}$$

where $\hat{\mathcal{L}}$ is defined by the communication topology.

From Lemma 2, one knows that there exists a positive definite matrix P such that (4) holds.

We consider the Lyapunov function $V_1 = \boldsymbol{\varepsilon}^{\rm T}P\boldsymbol{\varepsilon}$, whose derivative along system (21) is

$$\dot{V}_1|_{(21)} = -\frac{2}{L}\|\boldsymbol{\varepsilon}\|^2 + \frac{2}{L}\boldsymbol{\varepsilon}^{\rm T}P(I_N \otimes C)\boldsymbol{X} + 2\boldsymbol{\varepsilon}^{\rm T}P\boldsymbol{F}. \tag{22}$$

To get the estimation of the term $2\boldsymbol{\varepsilon}^{\rm T}P\boldsymbol{F}$, the following estimation of Δu_k and $\hat{\boldsymbol{f}}_k$, which are contained in $\boldsymbol{F}$, is needed.

From the definition of Δu_k in (15) and (19), we can get

$$\begin{aligned}|\Delta u_k| = &\left|\frac{1}{L^n}KH\left(\sum_{j=1}^{N} a_{kj}(\hat{\boldsymbol{x}}_k - \hat{\boldsymbol{x}}_j) + b_k(\hat{\boldsymbol{x}}_k - \hat{\boldsymbol{x}}_0)\right)\right| = \\ &\frac{1}{L^n}\|K\|\left|\left(\sum_{j=1}^{N} a_{kj}(\boldsymbol{\varepsilon}_k - \boldsymbol{\varepsilon}_j) + b_k\boldsymbol{\varepsilon}_k\right)\right| \le \\ &\frac{1}{L^n}\|K\|\|A_k\|\|\boldsymbol{\varepsilon}\|,\end{aligned}$$

where A_k is defined as $A_k = \alpha_k \otimes I_n$, with α_k being the kth row of matrix $\hat{\mathcal{L}}$.

To estimate $\hat{\boldsymbol{f}}_k$, we need to estimate $\hat{f}_{k,i}$ which is the ith element of $\hat{\boldsymbol{f}}_k$. For $i = 1, 2, \cdots, n-1$, we have

$$\begin{aligned}L^{i-1}|\hat{f}_{k,i}| = &L^{i-1}|f_i(t, \hat{x}_{k,i+2}, \cdots, \hat{x}_{k,n}, u_k) - \\ &f_i(t, \hat{x}_{0,i+2}, \cdots, \hat{x}_{0,n}, u_0)| \le \\ &\frac{c}{L^2}\sqrt{n}\|\boldsymbol{\varepsilon}_k\| + \frac{c}{L^2}\|K\|\|A_k\|\|\boldsymbol{\varepsilon}\|.\end{aligned}$$

Using the estimation for $L^{i-1}\hat{f}_{k,i}$, we can get the estimation of $\|\boldsymbol{F}\|$ as follows

$$\begin{aligned}\|\boldsymbol{F}\|^2 = &\sum_{k=1}^{N}\|\boldsymbol{F}_k\|^2 \le \\ &\sum_{k=1}^{N} n\left(\frac{c}{L^2}\sqrt{n}\|\boldsymbol{\varepsilon}_k\| + \frac{c}{L^2}\|K\|\|A_k\|\|\boldsymbol{\varepsilon}\|\right)^2 \le \\ &\sum_{k=1}^{N}\left(\frac{2c^2}{L^4}n^2\|\boldsymbol{\varepsilon}_k\|^2 + \frac{2c^2}{L^4}n\|K\|^2\|A_k\|^2\|\boldsymbol{\varepsilon}\|^2\right) \le \\ &\frac{\beta^2}{L^4}\|\boldsymbol{\varepsilon}\|^2,\end{aligned} \tag{23}$$

where β is dependent on the known constants $\|A_k\|$, $\|K\|$, c, n and N.

From (22) and (23), one obtains

$$\begin{aligned}\dot{V}_1|_{(21)} \le &-\frac{2}{L}\|\boldsymbol{\varepsilon}\|^2 + \frac{2}{L}\|\boldsymbol{\varepsilon}\|\|P\|\|I_N \otimes C\|\|\boldsymbol{X}\| + \\ &2\|\boldsymbol{\varepsilon}\|\|P\|\|\boldsymbol{F}\| \le \\ &-\frac{2}{L}\|\boldsymbol{\varepsilon}\|^2 + \|P\|\|I_N \otimes C\|(\frac{1}{L^2}\|\boldsymbol{\varepsilon}\|^2 + \|\boldsymbol{X}\|^2) + \\ &\frac{2\beta}{L^2}\|P\|\|\boldsymbol{\varepsilon}\|^2 \le \\ &-\frac{2}{L}\|\boldsymbol{\varepsilon}\|^2 + \frac{1}{L^2}(\|I_N \otimes C\| + 2\beta)\|P\|\|\boldsymbol{\varepsilon}\|^2 + \\ &\|P\|\|I_N \otimes C\|\|\boldsymbol{X}\|^2.\end{aligned} \tag{24}$$

Theorem 2. Under Assumptions 1$\sim$3, the consensus problem of the multi-agent system (1) can be solved by the dynamic output consensus protocol of the form (14) and (15).

Proof. We choose the Lyapunov function as

$$V = V_1 + \sum_{k=0}^{N} r_k\bar{V}_k,$$

where V_1 and $\bar{V}_k$ have been defined before, r_k $(k = 0, 1, \cdots, N)$ are positive constants to be decided later.

With (18) and (24), the derivative of V has the following calculation

$$\begin{aligned}\dot{V}|_{(16),(21)} = &\dot{V}_1 + \sum_{k=0}^{N} r_k\dot{\bar{V}}_k \le \\ &\sum_{k=0}^{N}\left(-\frac{r_k}{L}\|\boldsymbol{X}_k\|^2 + r_k\frac{2nc\|\bar{P}\|}{L^2}\|\boldsymbol{X}_k\|^2\right) - \frac{2}{L}\|\boldsymbol{\varepsilon}\|^2 + \\ &\frac{1}{L^2}(\|I_N \otimes C\| + 2\beta)\|P\|\|\boldsymbol{\varepsilon}\|^2 + \|P\|\|I_N \otimes C\|\|\boldsymbol{X}\|^2.\end{aligned}$$

If we choose

$$\begin{aligned}&L \ge \max\{\|P\|(\beta + \frac{1}{2}\|I_N \otimes C\|), 2nc\|\bar{P}\|, 1\} + \alpha_2, \quad \alpha_2 > 0, \\ &r_k \ge \frac{2}{L^{n-2}}\|P\|\|I_N \otimes C\| + \alpha_2, \qquad k = 1, 2, \cdots, N, \\ &r_0 \ge \frac{2N}{L^{n-2}}\|P\|\|I_N \otimes C\| + \alpha_2,\end{aligned}$$

then, we can get

$$\dot{V}|_{(16),(21)} \le -\frac{\alpha_2}{L^2}\|\boldsymbol{\varepsilon}\|^2 - \sum_{k=0}^{N}\frac{\alpha_2}{L^2}\|\boldsymbol{X}_k\|^2.$$

Thus, the state $\hat{\boldsymbol{x}}_k - \hat{\boldsymbol{x}}_0$ of the closed-loop system (1) and (14) with protocol (15) is asymptotically stable at $\hat{\boldsymbol{x}}_k - \hat{\boldsymbol{x}}_0 = 0$.

Noticing the fact that L satisfies $L \ge \max\{2nc\|\bar{P}\|, 1\} + \alpha_2$, and using (22), one can get

$$\dot{\bar{V}}_k|_{(16)} \le -\frac{\alpha_2}{L^2}\|\boldsymbol{X}_k\|^2,$$

which indicates $\boldsymbol{X}_k$ is asymptotically stable at $\boldsymbol{X}_k = 0$. With the definition of $\boldsymbol{X}_k$ in (16), $\tilde{\boldsymbol{x}}_k$ is also asymptotically stable at $\tilde{\boldsymbol{x}}_k = 0$.

As $\boldsymbol{x}_k - \boldsymbol{x}_0 = \hat{\boldsymbol{x}}_k - \hat{\boldsymbol{x}}_0 - \tilde{\boldsymbol{x}}_k + \tilde{\boldsymbol{x}}_0$, with the knowledge that the terms $\hat{\boldsymbol{x}}_k - \hat{\boldsymbol{x}}_0$, $\tilde{\boldsymbol{x}}_k$, and $\tilde{\boldsymbol{x}}_0$ are all asymptotically stable under protocol (14) and (15), it is certain that the global asymptotic stability of term $\boldsymbol{x}_k - \boldsymbol{x}_0$ is guaranteed. Then the protocol (14) and (15) can be used to solve the consensus reaching problem of the multi-agent system (1).

□

Remark 5. It is obvious that the observer of every agent used in our paper can only use the output signal of its own agent. This is much simpler and more efficient to design the observers for multi-agent system.

IV. A NUMERICAL EXAMPLE

To illustrate the designed protocols, the following numerical example is presented.

We consider a group of $3+1$ agents with the identical nonlinear dynamics, which are indexed by $0,1,2,3$. In this multi-agent system, the agent indexed by 0 is referred as the leader, and the agents indexed by $1,2,3$ are called followers. We also assume that all the follower agents know the input of the leader. For $k=0,1,2,3$, agent k is described by

$$\begin{cases} \dot{x}_{k,1} = x_{k,2} + \frac{1}{5+\sin^2 u_k} x_{k,3} + \frac{1}{6}\sin(u_k), \\ \dot{x}_{k,2} = x_{k,3} + \frac{1}{5+2\mathrm{e}^{-t}} u_k, \\ \dot{x}_{k,3} = u_k, \\ y_k = x_{k,1}. \end{cases} \tag{25}$$

The communication topology graph is shown in Fig. 1.

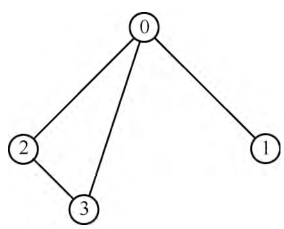


Fig. 1. Communication topology.

The adjacency matrix and degree matrix of the follower agents are denoted by $\mathcal{A}$ and $\mathcal{B}$, respectively. From Fig. 1, $\mathcal{A}$ and $\mathcal{B}$ can be defined as follows

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}.$$

It is easy to verify that Assumptions 1∼3 hold for this multi-agent system, and $c=0.2$.

First, the state feedback consensus protocol provided in Section III-A has the form of

$$u_k = KH\left(\sum_{j=1}^{3} a_{kj}(\boldsymbol{x}_k - \boldsymbol{x}_j) + b_k(\boldsymbol{x}_k - \boldsymbol{x}_0)\right) + u_0, \tag{26}$$

where $K=[-0.2,-0.7,-0.6]$, $H=\mathrm{diag}\{\frac{1}{L^3},\frac{1}{L^2},\frac{1}{L}\}$, with $L=10$, a_{kj} is the element in the kth row and jth column of matrix $\mathcal{A}$, and b_k is the kth diagonal element of matrix $\mathcal{B}$.

Fig. 2 shows the state responses of the closed-loop system consisting of (25) and (26), with the initial condition that

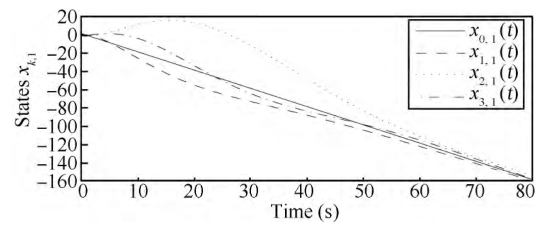
$$\boldsymbol{x}_0(0) = [2,-2,0]^{\mathrm{T}}, \quad \boldsymbol{x}_1(0) = [1,0,-1]^{\mathrm{T}},$$
$$\boldsymbol{x}_2(0) = [0,-1,1]^{\mathrm{T}}, \quad \boldsymbol{x}_3(0) = [-1,1,0]^{\mathrm{T}},$$

and $u_0(t)=0$.

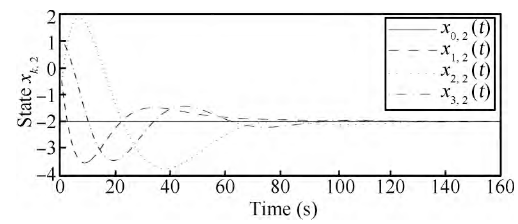
It can be seen that the signals of all follower agents do asymptotically tend to the states of the leader in Fig. 3, which shows the convergence of errors between the signals of the leader and followers.

Second, we consider the output feedback consensus problem. Using the method in Section III-B, the observer for agent $k(k=0,1,2,3)$ can be designed as

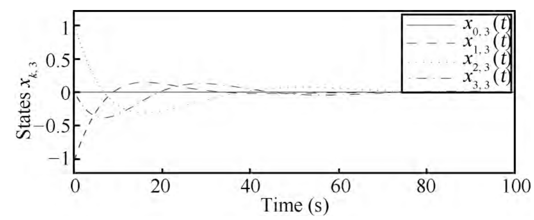
$$\begin{cases} \dot{\hat{x}}_{k,1} = \hat{x}_{k,2} + \frac{1}{5+\sin^2(u_k)}\hat{x}_{k,3} + \frac{1}{6}\sin(u_k) + \frac{1.1}{L}(x_{k,1}-\hat{x}_{k,1}), \\ \dot{\hat{x}}_{k,2} = \hat{x}_{k,3} + \frac{1}{5+2\mathrm{e}^{-t}}u_k + \frac{0.8}{L^2}(x_{k,1}-\hat{x}_{k,1}), \\ \dot{\hat{x}}_{k,3} = u_k + \frac{0.1}{L^3}(x_{k,1}-\hat{x}_{k,1}). \end{cases} \tag{27}$$



(a) The states $x_{k,1}$



(b) The states $x_{k,2}$



(c) The states $x_{k,3}$

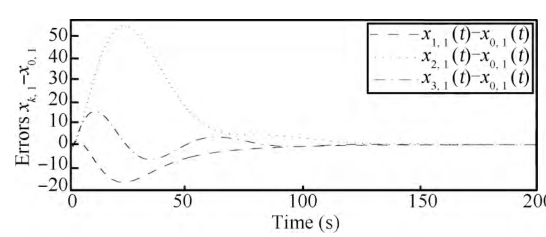
Fig. 2. The state responses of the closed-loop system with state consensus protocol.

Then, for $k=1,2,3$, the input u_k of each follower agent has the following form

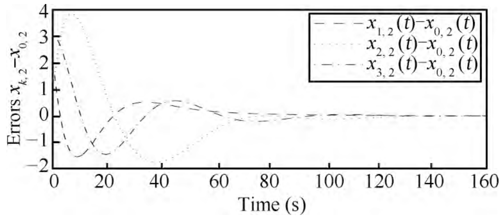
$$u_k = \frac{1}{L^3}KH\left(\sum_{j=1}^{3} a_{kj}(\hat{\boldsymbol{x}}_k - \hat{\boldsymbol{x}}_j) + b_k(\hat{\boldsymbol{x}}_k - \hat{\boldsymbol{x}}_0)\right), \tag{28}$$

where $\hat{x}_{i,j}$ is defined in (27), a_{kj} is the element in the kth row and jth column of the matrix $\mathcal{A}$, b_k is the kth diagonal element of the matrix $\mathcal{B}$, $K=[-0.2,-0.7,-0.6]$, and $H=\mathrm{diag}\{\frac{1}{L^3},\frac{1}{L^2},\frac{1}{L}\}$, with $L=20$.

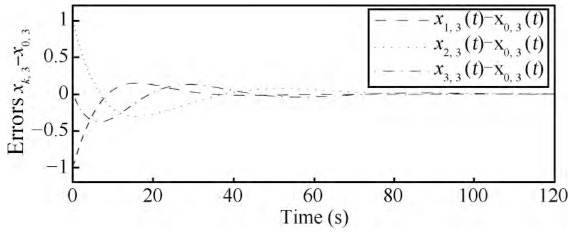
We also set the initial condition as before and the initial observer signals to be zero. Figs. 4 and 5 show the states, the errors of the signals between the leader and followers of the closed-loop system consisting of (25), (27) and (28), respectively.



(a) The errors $x_{k,1}-x_{0,1}$

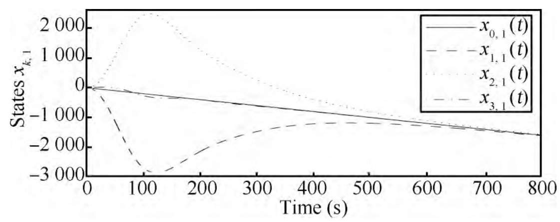


(b) The errors $x_{k,2} - x_{0,2}$

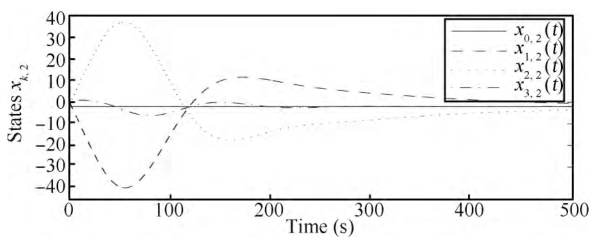


(c) The errors $x_{k,3} - x_{0,3}$

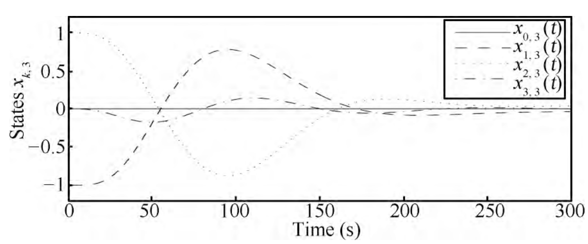
Fig. 3. The error responses of the closed-loop system with state consensus protocol.



(a) The states $x_{k,1}$

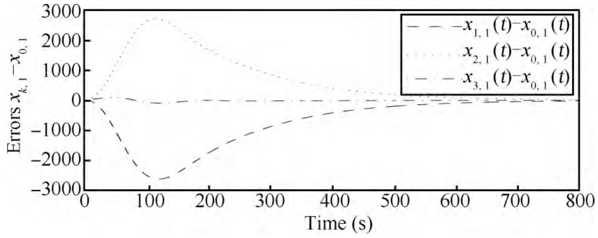


(b) The states $x_{k,2}$

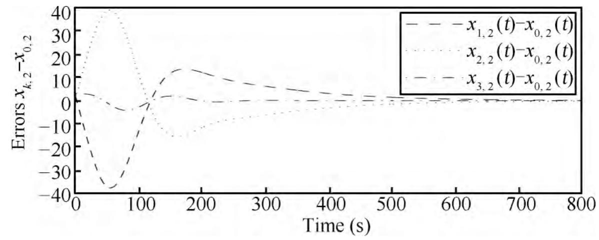


(c) The states $x_{k,3}$

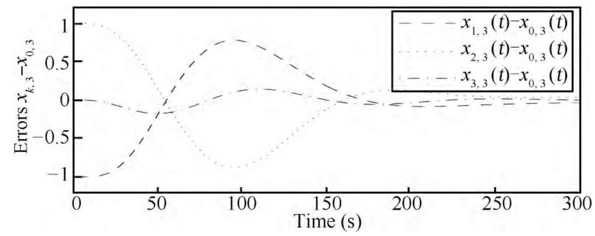
Fig. 4. The states responses of the closed-loop system with output consensus protocol.



(a) The errors $x_{k,1} - x_{0,1}$



(b) The errors $x_{k,2} - x_{0,2}$



(c) The errors $x_{k,3} - x_{0,3}$

Fig. 5. The error responses of the closed-loop system with output consensus protocol.

V. CONCLUSIONS

In this paper, we study the leader-follower consensus problem for a class of nonlinear multi-agent systems. Each agent has the identical nonlinear form and is coupled with an undirected communication topology. Different from [7], in our work, the input of the kth agent can be included in all nonlinear terms of the kth agent. Under the condition of undirected communication, we construct two consensus protocols, full state and dynamic output consensus protocol, for a class of nonlinear multi-agent systems.

REFERENCES

[1] Ma C Q, Li T, Zhang J F. Consensus control for leader-following multi-agent systems with measurement noises. *Journal of Systems Science and Complexity*, 2010, **23**(1): 35−49

[2] Seo J H, Shim H, Back J. Consensus of high-order linear systems using dynamic output feedback compensator: low gain approach. *Automatica*, 2009, **45**(11): 2659−2664

[3] Wang J H, Cheng D Z, Hu X M. Consensus of multi-agent linear dynamic systems. *Asian Journal of Control*, 2008, **10**(2): 144−155

[4] Li Q, Jiang Z P. Global analysis of multi-agent systems based on Vicsek′s model. *IEEE Transactions on Automatic Control*, 2009, **54**(12): 2876−2881

[5] Hong Y Q, Hu J P, Gao L X. Tracking control for multi-agent consensus with an active leader and variable topology. *Automatica*, 2006, **42**(7): 1177−1182

[6] Li Z, Liu X, Fu M, Xie L. Global H_∞ consensus of multi-agent systems with Lipschitz non-linear dynamics. *IET Control Theory & Applications*, 2012, **6**(13): 2041−2048

[7] Wang X H, Ji H B. Leader-follower consensus for a class of nonlinear multi-agent systems. *International Journal of Control, Automation and Systems*, 2012, **10**(1): 27−35

[8] Zhu W, Cheng D Z. Leader-following consensus of second-order agents with multiple time-varying delays. *Automatica*, 2010, **46**(12): 1994−1999

[9] Li W X, Chen Z Q. Leader-following consensus of second-order multi-agent systems with time-delay and nonlinear dynamics. In: Proceedings of the 31st Chinese Control Conference. Hefei, China: IEEE, 2012. 6124−6128

[10] Ma C Q, Zhang J F. Necessary and sufficient conditions for consensusability of linear multi-agent systems. *IEEE Transactions on Automatic Control*, 2010, **55**(5): 1263−1268

[11] Qin J H, Gao H J, Zheng W X. Second-order consensus for multi-agent systems with switching topology and communication delay. *Systems & Control Letters*, 2011, **60**(6): 390−397

[12] Huang Q Z. Consensus analysis of multi-agent discrete-time systems. *Acta Automatica Sinica*, 2012, **38**(7): 1127−1133

[13] Yu H W, Zheng Y F, Li D G. Consensus behavior of multi-agent systems under digital network topology. *Acta Automatica Sinica*, 2012, **38**(3): 357−363

[14] Yan J, Guan X P, Luo X Y, Yang X. Consensus and trajectory planning with input constraints for multi-agent systems. *Acta Automatica Sinica*, 2012, **38**(7): 1074−1082

[15] Wang Z, Zhang W L, Guo Y. Adaptive output consensus tracking of uncertain multi-agent systems. In: Proceedings of American Control Conference. San Francisco, CA, USA: IEEE, 2011. 3387−3392

[16] Yoo S J. Distributed adaptive containment control of uncertain nonlinear multi-agent systems in strict-feedback form. *Automatica*, 2013, **49**(7): 2145−2153

[17] Zhang X F, Boukas E K, Liu Y G, Baron L. Asymptotic stabilization of high-order feedforward systems with delays in the input. *International Journal of Robust and Nonlinear Control*, 2010, **20**(12): 1395−1406

[18] Zhang X F, Baron L, Liu Q R, Boukas E. Design of stabilizing controllers with a dynamic gain for feedforward nonlinear time-delay systems. *IEEE Transactions on Automatic Control*, 2011, **56**(3): 692−697

[19] Zhang X F, Liu Q R, Baron L, Boukas E. Feedback stabilization for high order feedforward nonlinear time-delay systems. *Automatica*, 2011, **47**(5): 962−967

[20] Zhang X, Gao H, Zhang C. Global asymptotic stabilization of feedforward nonlinear systems with a delay in the input. *International Journal of Systems Science*, 2006, **37**(3): 141−148

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