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International Journal of Control

ISSN: 0020-7179 (Print) 1366-5820 (Online) Journal homepage: www.tandfonline.com/journals/tcon20

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To cite this article: Le Chang, Chenghui Zhang, Xianfu Zhang & Xiandong Chen (2017) Decentralised regulation of nonlinear multi-agent systems with directed network topologies, International Journal of Control, 90:11, 2338-2348, DOI: 10.1080/00207179.2016.1245868

To link to this article: https://doi.org/10.1080/00207179.2016.1245868

Published online: 02 Dec 2016.

INTERNATIONAL JOURNAL OF CONTROL, 2017
VOL. 90, NO. 11, 2338–2348
http://dx.doi.org/10.1080/00207179.2016.1245868

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Decentralised regulation of nonlinear multi-agent systems with directed network topologies

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ABSTRACT

This paper aims to address the leader–follower regulation problem of multi-agent systems with directed network topologies, where the agents are described by feedforward nonlinearities with the growth rate being unknown a priori. Both the state feedback regulation protocol and the output feedback regulation protocol are delicately constructed such that all the states of followers can converge to the leader state globally. In this paper, a model transformation is firstly performed and the leader–follower regulation problem can be transformed into a general regulation problem. Then, by introducing an appropriate state transformation, the regulation problem can be changed into a parameter determined problem. It is proved that the parameter can be determined by both the properties of M-matrices and the estimates of nonlinear terms. Finally, a numerical example is presented to show the feasibility of designed protocols.

ARTICLE HISTORY
Received 5 January 2016
Accepted 3 October 2016

KEYWORDS
Multi-agent system; low-gain feedback; leader–follower regulation; feedforward nonlinear system

1. Introduction

The multi-agent system has drawn much attention from researchers in recent years (see e.g. Lynch, 1996; Peng and Yang, 2009), due to its broad range of applications such as unmanned air vehicles, microgrids, mobile robots and multiple agents. Cao, Yu, Ren, and Chen (2013) gave an overview of papers about the distributed coordination of multiple vehicles, and summarised some research directions including the study of consensus. The regulation problem for multi-agent systems is a generalisation of the leader–follower consensus problem (see e.g. Su & Huang, 2014), and has recently received more and more attention (see e.g. Li, Duan, Xie, & Liu, 2012; Wang, Ni, & Ma, 2015).

Recently, the considered problem is rising for the nonlinear multi-agent systems as the nonlinearity is ubiquitous in physical phenomena. Specifically, some researchers have been focusing on the cases when the agents can be described as the strict feedback forms. Wang and Ji (2012) studied the consensus protocol for multi-agent systems, where the feedback dynamic only can satisfy Lipschitz conditions with known constant gains. Yoo (2013) developed both the function approximation technique and the recursive computation method to study the tracking problem for uncertain feedback multi-agent systems. Zhang, Liu, and Feng (2015) presented the dynamic high-gain method to study the time-varying multi-agent system, where the designed controller can maintain a time-varying gain. Liu and Liang (2016) studied the finite-time consensus problem for the feedback multi-agent system, where the constructed algorithm can render the considered system consensus in a finite time.

However, little results about the multi-agent system have been published for the feedforward cases which are widely considered in the classical nonlinear problem. To fill the gap, we elaborate the regulation results for directed multi-agent systems, where the agents are represented as unknown feedforward dynamics. It is noted that we mainly focus on studying in the theoretical field of feedforward systems, which is analogous to the study of feedback multi-agent systems (see e.g. Liu & Liang, 2016; Wang & Ji, 2012; Yoo, 2013; Zhang et al., 2015). Meanwhile, feedforward systems can represent many physical systems (see e.g. Tan, Lai, Tse, & Cheung, 2006), which gives certain practical significance to this result.

To solve this considered problem, a decentralised dynamic low-gain technology is developed to construct both the state feedback protocol and the output feedback protocol. Our main contributions are summarised below.

- The low-gain technology is developed to study uncertain feedforward systems, which can not be studied by the classical technology, such as the forward technology, the adding integrations technology, and the saturated controls technology.

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Moreover, the study of uncertain feedforward systems is more challenging and difficult than that of feedback cases as the low gain is difficult to tackle the uncertain growth. Specifically, the rate of the dynamic gain is determined to dominant the uncertain feedforward systems, while the dynamic gain without considering its rate was chosen by Qian and Lin (2003) for classical uncertain feedback systems.

- We consider the multi-agent system with feedforward dynamics in the directed network. Up to now, to the best of our knowledge, there is no result focusing on directed feedforward multi-agent systems. Moreover, the feedforward dynamic satisfies a very general growth condition and the precise knowledge of the growth gain is not required to be known a priori. This is a generation of our former work (see Zhang, Chang, & Zhang, 2014), where the agents in an undirected network are represented as feedforward dynamics with no uncertainties.

This paper is organised as follows. Section 2 formulates the considered problem and gives some basic Lemmas. Section 3 presents the main approaches to design the regulation protocols. A numerical example is presented in Section 4. Section 5 gives some concluding remarks.

In this paper, I denotes the unit matrix of the corresponding dimension, and the arguments of functions are sometimes simplified, for instance, a function $f(x(t))$ is denoted by $f(x)$.

2. Problem formulation

In this part, we describe the problem of this paper, and present some basic Lemmas which are essential to the proof of our main results. Without loss of generality, the leader is labelled as 1, while the followers are labelled as 2, 3, …, N. The motion of each agent is described as

$$\begin{aligned} \dot{x}_{k,i} &= x_{k,i+1} + f_i(t, \bar{x}_{k,i+1}, u_k),\ i = 1, 2, \ldots, n-1 \\ \dot{x}_{k,n} &= u_k \\ y_k &= x_{k,1}, \quad k = 1, 2, \ldots, N. \end{aligned} \tag{1}$$

where $\bar{x}_{k,i+1} = [x_{k,i+2}, x_{k,i+3}, \ldots, x_{k,n}]^T \in \mathbb{R}^{n-i-1}$, $x_k = [x_{k,1}, x_{k,2}, \ldots, x_{k,n}]^T \in \mathbb{R}^n$ is the state of agent k, $u_k \in \mathbb{R}$ and $y_k \in \mathbb{R}$ are the input and output of agent k, respectively. We assume that the continuous functions $f_i(\cdot)$, $i = 1, 2, \ldots, n-1$, in system (1), satisfy the following Assumption.

Assumption 2.1: *For $i = 1, 2, \ldots, n-1$, and any $(t, \bar{x}_{k,i+1}, u_k)$, $(t, \bar{x}_{1,i+1}, u_1) \in \mathbb{R}^+ \times \mathbb{R}^{n-i-1} \times \mathbb{R}$, $k = 2, 3, \ldots, N$, the following inequality holds:*

$$\begin{aligned} &\left| f_i\left(t, \bar{x}_{k,i+1}, u_k\right) - f_i\left(t, \bar{x}_{1,i+1}, u_1\right)\right| \\ &\quad \le \delta \sum_{j=i+2}^{n+1} \left| x_{k,j} - x_{1,j}\right|, \end{aligned} \tag{2}$$

where $x_{k,n+1} = u_k$, $x_{1,\,n+1} = u_1$, and δ is an unknown nonnegative constant.

Remark 2.1: This considered nonlinear terms are affected not only by the measurement input, but also by the uncertain terms. It is noted that the agent dynamic is commonly considered in the study of classical nonlinear systems when δ is a known constant. However, there are few results considering the feedforward dynamic when δ is an unknown constant.

To simplify the description, the dynamic of agent k can also be rewritten as the following matrix form:

$$\begin{aligned} \dot{x}_k &= Ax_k + Bu_k + F_k, \\ y_k &= Cx_k, \end{aligned} \tag{3}$$

where $A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$,

$F_k = \begin{bmatrix} f_1\left(t, \bar{x}_{k,2}, u_k\right) \\ f_2\left(t, \bar{x}_{k,3}, u_k\right) \\ \vdots \\ f_{n-1}\left(t, u_k\right) \\ 0 \end{bmatrix}$, and $C = [1, 0, \ldots, 0, 0]$.

Meanwhile, the communication topology of N agents is denoted as a graph $\mathcal{G}$ and a weight matrix $\mathcal{A}$ with the leader being the root. It is necessary to assume that the graph $\mathcal{G}$ contains a directed spanning tree and all the followers know the input of the leader agent, which is commonly considered in the study of multi-agent systems, such as the researches considered in Wen, Peng, Rahmani, and Yu (2014), Meng, Jia, and Du (2015), and Yoo (2013). Based on the definition of the Laplacian matrix $\mathcal{L}$ (see e.g. Wen, Hu, Yu, Cao, & Chen, 2013), one can easily get that the Laplacian matrix $\mathcal{L}$ has the following form:

$$\mathcal{L} = \begin{bmatrix} 0 & 0 \\ \eta & \hat{\mathcal{L}} \end{bmatrix}, \tag{4}$$

with $\eta \in \mathbb{R}^{N-1}$ being a non-zero column vector, and $\hat{\mathcal{L}} \in \mathbb{R}^{(N-1)\times(N-1)}$ being an M-matrix.

The objective of this paper is to design the protocol u_k, $k = 2, 3, \ldots, N$, for each follower based on its neighbours' information, such that the leader–follower regulation of multi-agent system (3), (4) is achieved, that is,

$$\lim_{t\to+\infty} \|x_k(t) - x_1(t)\| = 0, \quad k = 2, 3, \dots, N.$$

Remark 2.2: The external input u_1 is reasonable to be existed in the multi-agent system, which has been widely considered in the existing researches (see e.g. Wang & Ji, 2012; Wen et al., 2013; Zhang et al., 2014, 2015). Moreover, the external input u_1 can represent a variable in the industrial process, such as the user requirement in micro-grids, or the man controlled leader in a group of autonomous agent including unmanned aerial vehicles, unmanned ground vehicles, and unmanned undersea vehicles.

Lemma 2.1 (Wen et al., 2013)**:** *Suppose that the directed graph $\mathcal{G}$ contains a directed spanning tree. Then, 0 is a simple eigenvalue of its Laplacian matrix $\mathcal{L}$, and the real part of all the other eigenvalues is positive.*

Lemma 2.2 (Berman & Plemmons, 1979; Wen et al., 2013)**:** *The following propositions are equivalent for a matrix $\hat{\mathcal{L}} = [l_{ij}]_{(N-1)\times(N-1)} \in \mathbb{R}^{(N-1)\times(N-1)}$.*

1. *$\hat{\mathcal{L}}$ is an M-matrix.*
2. *$l_{ij} \le 0$, $i \neq j$, and there exists a positive definite diagonal matrix $R = diag\{r_1, r_2, \dots, r_{N-1}\}$ such that $\hat{\mathcal{L}}R + R\hat{\mathcal{L}}^T > 0$.*
3. *$l_{ij} \le 0$, $i \neq j$, and each of its eigenvalues has the positive real part.*

Remark 2.3: From the second proposition of Lemma 2.2, one can achieve that there exists a positive definite matrix $R = \mathrm{diag}\{r_1, r_2, \dots, r_{N-1}\}$ such that $\hat{\mathcal{L}}R + R\hat{\mathcal{L}}^T$ is a positive definite matrix. Then, for a given matrix R, a positive constant η can be found to satisfy $\hat{\mathcal{L}}R + R\hat{\mathcal{L}}^T \ge \eta I$.

Lemma 2.3 (Su, Chen, Lam, & Lin, 2013)**:** *With the definition of A, B in (3), there exists a positive definite matrix P such that*

$$PA + A^T P - PBB^T P = -I. \tag{5}$$

Lemma 2.4 (Su et al., 2013)**:** *Suppose $D \in \mathbb{R}^{n\times n}$ is the matrix defined as $D = diag\{n, n-1, \dots, 1\}$, and $P \in \mathbb{R}^{n\times n}$ is a positive defined matrix. Then, a positive constant σ can be found such that*

$$PD + DP \ge \sigma I. \tag{6}$$

Lemma 2.5 (Zhang & Cheng, 2005)**:** *With the definition of A, C in (3), there exists a column vector $L = [l_1, l_2, \dots, l_n]^T$ such that the matrix $A - LC$ is a Hurwitz matrix.*

3. Main results

3.1 State feedback regulation protocol

We propose a constructive procedure to design the state feedback regulation protocol. For $k = 2, \dots, N$, the error state between agent k and leader 1 is defined as $\tilde{x}_{k,i} = x_{k,i} - x_{1,i}$, $i = 1, 2, \dots, n$. Apparently, one can obtain

$$\begin{aligned} \dot{\tilde{x}}_{k,i} &= \tilde{x}_{k,i+1} + f_i\left(t, \bar{x}_{k,i+1}, u_k\right) - f_i(t, \bar{x}_{1,i+1}, u_1), \\ &\quad i = 1, 2, \dots, n-1, \\ \dot{\tilde{x}}_{k,n} &= u_k - u_1, \quad k = 2, 3, \dots, N. \end{aligned} \tag{7}$$

Then, we introduce the input protocol form

$$u_k = \alpha B^T P \sum_{j=1}^{N} a_{kj}\Gamma(x_j - x_k) + u_1, \quad k = 2, 3, \dots, N, \tag{8}$$

where α is a constant to be determined later, B is as in (3), P is defined in Lemma 2.3, a_{ij} is the entry of the communication weight matrix $\mathcal{A}$, and $\Gamma = \mathrm{diag}\{\frac{1}{\gamma^n}, \frac{1}{\gamma^{n-1}}, \cdots, \frac{1}{\gamma}\}$ with $\gamma > 1$ being a function to be determined later.

The change of coordinate, for $k = 2, 3, \dots, N$,

$$\hat{x}_{k,i} = \frac{\tilde{x}_{k,i}}{\gamma^{n-i+1}}, \quad i = 1, 2, \dots, n,$$

can convert system (7) and input (8) into

$$\dot{\hat{x}}_k = \frac{1}{\gamma}A\hat{x}_k + \frac{1}{\gamma}B(u_k - u_1) + \tilde{F}_k - \frac{\dot{\gamma}}{\gamma}D\hat{x}_k, \tag{9}$$

and

$$u_k = -\alpha B^T P \sum_{j=2}^{N} l_{kj}\hat{x}_j + u_1, \tag{10}$$

where $\hat{x}_k = [\hat{x}_{k,1}, \hat{x}_{k,2}, \dots, \hat{x}_{k,n}]^T$, A is denoted in (3), D is defined in Lemma 2.4, l_{ij} is the entry of the matrix $\mathcal{L}$ in (4), and

$$\tilde{F}_k = \begin{bmatrix} \frac{1}{\gamma^n} f_1\left(t, \bar{x}_{k,2}, u_k\right) - \frac{1}{\gamma^n} f_1\left(t, \bar{x}_{1,2}, u_1\right) \\ \frac{1}{\gamma^{n-1}} f_2(t, \bar{x}_{k,3}, u_k) - \frac{1}{\gamma^{n-1}} f_2\left(t, \bar{x}_{1,3}, u_1\right) \\ \vdots \\ \frac{1}{\gamma^2} f_{n-1}(t, u_k) - \frac{1}{\gamma^2} f_{n-1}\left(t, u_1\right) \\ 0 \end{bmatrix}.$$

The closed-loop system (9), (10) can be rewritten as

$$\dot{\hat{x}} = \frac{1}{\gamma}I \otimes A\hat{x} - \frac{\alpha}{\gamma}\hat{\mathcal{L}} \otimes BB^T P\hat{x} + F - \frac{\dot{\gamma}}{\gamma}I \otimes D\hat{x}, \tag{11}$$

where $\hat{x} = [\hat{x}_2^T, \hat{x}_3^T, \ldots, \hat{x}_N^T]^T$, and $F = [\tilde{F}_2^T, \tilde{F}_3^T, \cdots, \tilde{F}_N^T]^T$.

With the description before, we state one of our main results below.

Theorem 3.1: *Under Assumption 2.1, constant α, and function γ can be chosen for the state feedback protocol (8) such that the state of multi-agent system (3), (4) will satisfy*

$$\lim_{t\to+\infty} \|x_k(t) - x_1(t)\| = 0, \quad k = 2, 3, \ldots, N.$$

Proof: It is obvious that we just need to determine parameter α and dynamic gain γ such that the state of closed-loop system (11) converge to $\hat{x} = 0$. Let $V = \hat{x}^T(R \otimes P)\hat{x}$, where R and P are defined in Lemma 2.2 and Lemma 2.3, respectively. From Lemma 2.2 and Lemma 2.3, we can know

$$PA + A^TP - PBB^TP = -I,$$

and

$$\hat{\mathcal{L}}R + R\hat{\mathcal{L}}^T \geq \eta I$$

with $\eta > 0$ being a known constant.

The derivative of V along (11) can be calculated as

$$\begin{aligned}\dot{V}|_{(11)} &= \tfrac{1}{\gamma}\hat{x}^T\big(R \otimes (PA + A^TP) \\ &\quad - \alpha(\hat{\mathcal{L}}R + R\hat{\mathcal{L}}^T) \otimes PBB^TP\big)\hat{x} \\ &\quad + 2\hat{x}^T(R \otimes P)F - \tfrac{\dot{\gamma}}{\gamma}\hat{x}^T R \otimes (PD + DP)\hat{x} \\ &\leq \lambda_{\max}(R)\tfrac{1}{\gamma}\hat{x}^T I \otimes (PA + A^TP)\hat{x} \\ &\quad - \alpha\eta\tfrac{1}{\gamma}\hat{x}^T I \otimes PBB^TP\hat{x} \\ &\quad + 2\hat{x}^T(R \otimes P)F - \tfrac{\dot{\gamma}}{\gamma}\sigma\lambda_{\min}(R)\|\hat{x}\|^2,\end{aligned}$$

where σ is defined in Lemma 2.4.

Suppose $\alpha = \lambda_{\max}(R)\frac{1}{\eta}$, and then

$$\begin{aligned}\dot{V}|_{(11)} &\leq \lambda_{\max}(R)\tfrac{1}{\gamma}\hat{x}^T I \otimes (PA + A^TP - PBB^TP)\hat{x} \\ &\quad + 2\hat{x}^T(R \otimes P)F - \tfrac{\dot{\gamma}}{\gamma}\sigma\lambda_{\min}(R)\|\hat{x}\|^2 \\ &\leq -\lambda_{\max}(R)\tfrac{1}{\gamma}\|\hat{x}\|^2 + 2\hat{x}^T(R \otimes P)F \\ &\quad - \tfrac{\dot{\gamma}}{\gamma}\sigma\lambda_{\min}(R)\|\hat{x}\|^2.\end{aligned} \tag{12}$$

To determine γ, we will estimate the norm of function F. According to (2) and the technology studied in Zhang et al. (2014), a positive constant β_1 can be found such that the following estimation holds,

$$\|F\| \leq \delta\frac{\beta_1}{\gamma^2}\|\hat{x}\|. \tag{13}$$

Thus, from (12) and (13), there exists a positive constant β_2 such that

$$\begin{aligned}\dot{V}|_{(11)} &\leq -\lambda_{\max}(R)\frac{1}{\gamma}\|\hat{x}\|^2 + \delta\frac{\beta_2}{\gamma^2}\|\hat{x}\|^2 \\ &\quad - \frac{\dot{\gamma}}{\gamma}\sigma\lambda_{\min}(R)\|\hat{x}\|^2.\end{aligned} \tag{14}$$

We can choose $\gamma = c_1 t + 1$ with c_1 being a positive constant to be determined. With the knowledge of this equation

$$\lambda_{\min}(P)\lambda_{\min}(R)\|\hat{x}\|^2 \leq V \leq \lambda_{\max}(P)\lambda_{\max}(R)\|\hat{x}\|^2,$$

one can achieve

$$\begin{aligned}\dot{V}|_{(11)} &\leq -\left(\lambda_{\max}(R)\tfrac{1}{\gamma} + \tfrac{c_1}{\gamma}\sigma\lambda_{\min}(R)\right)\|\hat{x}\|^2 + \delta\tfrac{\beta_2}{\gamma^2}\|\hat{x}\|^2 \\ &\leq -\left(\lambda_{\max}(R)\tfrac{1}{\gamma} + \tfrac{c_1}{\gamma}\sigma\lambda_{\min}(R)\right)\tfrac{1}{\lambda_{\max}(P)\lambda_{\max}(R)}V \\ &\quad + \delta\tfrac{\beta_2}{\gamma^2}\tfrac{1}{\lambda_{\min}(P)\lambda_{\min}(R)}V,\end{aligned}$$

which indicates that

$$\begin{aligned}V(t) &\leq \gamma^{-(\lambda_{\max}(R)+c_1\sigma\lambda_{\min}(R))\frac{1}{c_1\lambda_{\max}(P)\lambda_{\max}(R)}} \\ &\quad e^{\delta\beta_2\frac{1}{c_1\lambda_{\min}(P)\lambda_{\min}(R)}(1-\frac{1}{\gamma})}V(0).\end{aligned}$$

Moreover, since

$$\lambda_{\min}(P)\lambda_{\min}(R)\frac{1}{\gamma^{2n}}\|\tilde{x}(t)\|^2 \leq V(t),$$

we have

$$\begin{aligned}\|\tilde{x}(t)\|^2 &\leq \frac{1}{\lambda_{\min}(P)\lambda_{\min}(R)} \\ &\quad \gamma^{2n-(\lambda_{\max}(R)+c_1\sigma\lambda_{\min}(R))\frac{1}{c_1\lambda_{\max}(P)\lambda_{\max}(R)}} \\ &\quad e^{\delta\beta_2\frac{1}{c_1\lambda_{\min}(P)\lambda_{\min}(R)}\left(1-\frac{1}{\gamma}\right)}V(0).\end{aligned}$$

Note that c_1 can be chosen such that $c_1 \leq \frac{1}{2n\lambda_{\max}(P)}$, which shows that $\lim_{t\to+\infty}(c_1 t + 1)^{2n-(\lambda_{\max}(R)+c_1\sigma\lambda_{\min}(R))\frac{1}{c_1\lambda_{\max}(P)\lambda_{\max}(R)}} = 0$. With the knowledge of $\lim_{t\to+\infty}\delta\beta_2\frac{1}{\gamma} = 0$, we can achieve $\lim_{t\to+\infty}\|x_k(t) - x_1(t)\| = 0, \quad k = 2, 3, \ldots, N$.

That is, α, γ can be found in the protocol (8) to render system (1) leader–follower regulation. ∎

Remark 3.1: It is shown that function γ satisfies $\gamma \geq 1$ for all $t > 0$, which is result from $\dot{\gamma} \geq 0$. The technology in Zhang et al. (2014) makes the gain be a constant and the constant is larger then 1. Thus, the estimate (13) is reasonable to be achieve.

3.2 Output feedback regulation protocol

The regulation protocol designed before is not valid when the agent can not get all the information of its neighbour. Thus, it is necessary to design the output feedback regulation protocol, which will be constructed below.

For $k = 1, 2, \ldots, N$, the observer for agent k can be designed as,

$$\begin{cases} \dot{z}_{k,i} = z_{k,i+1} + \gamma^{-i} l_i (y_k - z_{k,1}), \ i = 1, 2, \ldots, n-1, \\ \dot{z}_{k,n} = u_k + \gamma^{-n} l_n (y_k - z_{k,1}), \end{cases} \tag{15}$$

where $z_{k,i} \in \mathbb{R}$ is the state, l_i is defined in Lemma 2.5, and γ is a function to be determined later.

Defining $\tilde{x}_{k,i} = x_{k,i} - x_{1,i}$, and $\tilde{z}_{k,i} = z_{k,i} - z_{1,i}$, $i = 1, 2, \ldots, n$, $k = 2, 3, \ldots, N$, one can achieve

$$\begin{cases} \dot{\tilde{x}}_{k,i} = \tilde{x}_{k,i+1} + f_i\left(t, \bar{x}_{k,i+1}, u_k\right) \\ \qquad\quad - f_i\left(t, \bar{x}_{1,i+1}, u_1\right), \quad i = 1, 2, \ldots, n-1, \\ \dot{\tilde{x}}_{k,n} = u_k - u_1, \quad k = 2, 3, \ldots, N, \end{cases} \tag{16}$$

and

$$\begin{cases} \dot{\tilde{z}}_{k,i} = \tilde{z}_{k,i+1} + \gamma^{-i} l_i (\tilde{x}_{k,1} - \tilde{z}_{k,1}), \quad i = 1, 2, \ldots, n-1, \\ \dot{\tilde{z}}_{k,n} = u_k - u_1 + \gamma^{-n} l_n (\tilde{x}_{k,1} - \tilde{z}_{k,1}), \ k = 2, 3, \ldots, N. \end{cases} \tag{17}$$

Let $\hat{x}_{k,i} = \frac{\tilde{x}_{k,i} - \tilde{z}_{k,i}}{\gamma^{n-i+1}}$, and $\hat{z}_{k,i} = \frac{\tilde{z}_{k,i}}{\gamma^{n-i+1}}$, $i = 1, 2, \ldots, n$, $k = 2, 3, \ldots, N$, and then we can rewrite the system into the matrix form

$$\dot{\hat{x}}_k = \frac{1}{\gamma}(A - LC)\hat{x}_k + \tilde{F}_k - \frac{\dot{\gamma}}{\gamma} D\hat{x}_k, \ k = 2, 3, \ldots, N, \tag{18}$$

and

$$\dot{\hat{z}}_k = \frac{1}{\gamma} A\hat{z}_k + \frac{1}{\gamma} B(u_k - u_1) - \frac{1}{\gamma} LC\hat{x}_k - \frac{\dot{\gamma}}{\gamma} D\hat{z}_k, \quad k = 2, 3, \ldots, N, \tag{19}$$

where $\hat{x}_k = [\hat{x}_{k,1}, \hat{x}_{k,2}, \ldots, \hat{x}_{k,n}]^T$, $\hat{z}_k = [\hat{z}_{k,1}, \hat{z}_{k,2}, \ldots, \hat{z}_{k,n}]^T$, A, B and C are denoted in (3), L is defined in Lemma 2.5, D is defined in Lemma 2.4, and

$$\tilde{F}_k = \begin{bmatrix} \frac{1}{\gamma^n} f_1\left(t, \bar{x}_{k,2}, u_k\right) - \frac{1}{\gamma^n} f_1\left(t, \bar{x}_{1,2}, u_1\right) \\ \frac{1}{\gamma^{n-1}} f_2(t, \bar{x}_{k,3}, u_k) - \frac{1}{\gamma^{n-1}} f_2\left(t, \bar{x}_{1,3}, u_1\right) \\ \vdots \\ \frac{1}{\gamma^2} f_{n-1}(t, u_k) - \frac{1}{\gamma^2} f_{n-1}\left(t, u_1\right) \\ 0 \end{bmatrix}.$$

Theorem 3.2: *With Assumption 2.1 holds, constant α, function γ can be chosen such that the nonlinear multi-agent system (3), (4) can achieve leader–follower regulation under the following protocol*

$$u_k = \alpha B^T P \sum_{j=1}^{N} a_{kj} \Gamma(z_j - z_k) + u_1, \ k = 2, 3, \ldots, N, \tag{20}$$

where B is denoted in (3), P is defined in Lemma 2.3, $\Gamma = \text{diag}\{\frac{1}{\gamma^n}, \frac{1}{\gamma^{n-1}}, \cdots, \frac{1}{\gamma}\}$, a_{ij} is the entry of the communication weight matrix $\mathcal{A}$, and $z_j = [z_{j,1}, z_{j,2}, \ldots, z_{j,n}]^T$ is defined in (15).

Proof: From the description before, this result can be deduced from the zero asymptotic convergence of systems (18) and (19).

Since $A - LC$ is a Hurwitz matrix, there exists a positive definite matrix P_1 such that

$$(A - LC)^T P_1 + P_1(A - LC) = -I, \tag{21}$$

and

$$DP_1 + P_1 D \geq \sigma_1 I \tag{22}$$

with $\sigma_1 > 0$ being a known constant.

Let $V_k = \hat{x}_k^T P_1 \hat{x}_k$, $k = 2, 3, \ldots N$. From (21) and (22), the derivative of V_k along (18) can be expressed as

$$\begin{aligned} \dot{V}_k|_{(18)} &= \hat{x}_k^T P_1 (\tfrac{1}{\gamma}(A - LC)\hat{x}_k + \tilde{F}_k) \\ &\quad + (\tfrac{1}{\gamma}(A - LC)\hat{x}_k + \tilde{F}_k)^T P_1 \hat{x}_k \\ &\quad - \tfrac{\dot{\gamma}}{\gamma} \hat{x}_k^T (P_1 D + DP_1)\hat{x}_k \\ &= -\tfrac{1}{\gamma}\|\hat{x}_k\|^2 + 2\hat{x}_k^T P_1 \tilde{F}_k - \sigma_1 \tfrac{\dot{\gamma}}{\gamma}\|\hat{x}_k\|^2. \end{aligned} \tag{23}$$

According to (2) and (18), constant β_1 can be found such that the following estimates hold,

$$\left\|\tilde{F}_k\right\| \leq \beta_1 \delta \frac{1}{\gamma^2}(\|\hat{x}_k\| + \|\hat{z}_k\|), \quad k = 2, 3, \ldots, N. \tag{24}$$

From (23) and (24), one can find constant β_2 such that

$$\dot{V}_k|_{(18)} \leq -\frac{1}{\gamma}\|\hat{x}_k\|^2 + \beta_2 \delta \frac{1}{\gamma^2}(\|\hat{x}_k\|^2 + \|\hat{z}_k\|^2) - \sigma_1 \frac{\dot{\gamma}}{\gamma}\|\hat{x}_k\|^2. \tag{25}$$

Meanwhile, based on (20), system (19) can be rewritten as the following compact form

$$\dot{\hat{z}} = \frac{1}{\gamma} I \otimes A\hat{z} - \alpha \frac{1}{\gamma} \hat{\mathcal{L}} \otimes BB^T P\hat{z} - \frac{1}{\gamma} I \otimes LC\hat{x} - \frac{\dot{\gamma}}{\gamma} I \otimes D\hat{z} \tag{26}$$

with $\hat{z} = [\hat{z}_2^T, \hat{z}_3^T, \ldots, \hat{z}_N^T]^T$, $\hat{x} = [\hat{x}_2^T, \hat{x}_3^T, \ldots, \hat{x}_N^T]^T$, and $\hat{\mathcal{L}}$ being the Laplacian matrix defined in (4).

Let $V_1 = \hat{z}^T (R \otimes P)\hat{z}$, where R and P are, respectively, defined in Lemma 2.2 and Lemma 2.3 satisfy

$$PA + A^T P - PBB^T P = -I,$$

$$PD + DP \geq \sigma_2 I,$$

and

$$\hat{\mathcal{L}}R + R\hat{\mathcal{L}}^T \geq \eta I$$

with $\eta > 0$, $\sigma_2 > 0$ being known constants.

Setting $\alpha = \lambda_{\max}(R)\frac{1}{\eta}$, a similar calculation shows that the derivative of V_1 along (26) holds

$$\begin{aligned} \dot{V}_1|_{(26)} = {} & \tfrac{1}{\gamma}\hat{z}^T \big(R \otimes (PA + A^T P) \\ & - \alpha(\hat{\mathcal{L}}R + R\hat{\mathcal{L}}^T) \otimes PBB^T P\big)\hat{z} \\ & -2\tfrac{1}{\gamma}\hat{z}^T (R \otimes PLC)\hat{x} - \tfrac{\dot{\gamma}}{\gamma}\hat{z}^T R \otimes (PD + DP)\hat{z} \\ \leq {} & -\lambda_{\max}(R)\tfrac{1}{\gamma}\|\hat{z}\|^2 + 2\beta_3 \tfrac{1}{\gamma}\|\hat{x}\|\|\hat{z}\| \\ & - \tfrac{\dot{\gamma}}{\gamma}\sigma_2 \lambda_{\min}(R)\|\hat{z}\|^2 \\ \leq {} & -\lambda_{\max}(R)\tfrac{1}{2\gamma}\|\hat{z}\|^2 + \beta_4 \tfrac{1}{\gamma}\|\hat{x}\|^2 - \tfrac{\dot{\gamma}}{\gamma}\sigma_2\lambda_{\min}(R)\|\hat{z}\|^2, \end{aligned} \tag{27}$$

where $\beta_3 = \|R \otimes PLC\|$, and $\beta_4 = 2\beta_3 \frac{1}{\lambda_{\max}(R)}$.

Note that $\|\hat{z}\|^2 = \sum_{k=2}^{N} \|\hat{z}_k\|^2$, and $\|\hat{x}\|^2 = \sum_{k=2}^{N} \|\hat{x}_k\|^2$. With $V = V_1 + 2\beta_4 \sum_{k=2}^{N} V_k$, a calculation followed (25) and (27) can be achieved

$$\begin{aligned} \dot{V}|_{(18),(26)} \leq {} & -\lambda_{\max}(R)\tfrac{1}{2\gamma}\|\hat{z}\|^2 - \beta_4 \tfrac{1}{\gamma}\|\hat{x}\|^2 \\ & + 2\beta_4\beta_2\delta\tfrac{1}{\gamma^2}\left(\|\hat{x}\|^2 + \|\hat{z}\|^2\right) \\ & -2\beta_4\sigma_1\tfrac{\dot{\gamma}}{\gamma}\|\hat{x}\|^2 - \tfrac{\dot{\gamma}}{\gamma}\sigma_2\lambda_{\min}(R)\|\hat{z}\|^2 \\ \leq {} & \Bigg(-\frac{1}{2\gamma}\min\{\lambda_{\max}(R), 2\beta_4\} + 2\beta_4\beta_2\delta\frac{1}{\gamma^2} \\ & - \frac{\dot{\gamma}}{\gamma}\min\{2\beta_4\sigma_1, \sigma_2\lambda_{\min}(R)\}\Bigg)(\|\hat{x}\|^2 + \|\hat{z}\|^2). \end{aligned}$$

Furthermore, based on the definition of V, one can get

$$\mu_1(\|\hat{x}\|^2 + \|\hat{z}\|^2) \leq V \leq \mu_2(\|\hat{x}\|^2 + \|\hat{z}\|^2).$$

where $\mu_1 = \min\{\lambda_{\min}(R)\lambda_{\min}(P),\ 2\beta_4\lambda_{\min}(P_1)\}$, and $\mu_2 = \max\{\lambda_{\max}(R)\lambda_{\max}(P), 2\beta_4\lambda_{\max}(P_1)\}$.

Therefore, positive constants β_5, β_6, and β_7 can be found such that

$$\dot{V}|_{(18),(26)} \leq -\beta_5 \frac{1}{\gamma} V + \delta\beta_6 \frac{1}{\gamma^2} V - \beta_7 \frac{\dot{\gamma}}{\gamma} V.$$

Letting $\gamma = c_1 t + 1$ with c_1 being a positive constant to be determined, we have

$$V(t) \leq \gamma^{-\frac{1}{c_1}\beta_5 - \beta_7} e^{\delta\beta_6 \frac{1}{c_1}(1-\frac{1}{\gamma})} V(0),$$

and

$$\|\tilde{x} - \tilde{z}\|^2 + \|\tilde{z}\|^2 \leq \frac{1}{\mu_1}\gamma^{2n-\frac{1}{c_1}\beta_5-\beta_7} e^{\delta\beta_6\frac{1}{c_1}(1-\frac{1}{\gamma})} V(0),$$

where $\tilde{x} = [\tilde{x}_2^T, \tilde{x}_3^T, \ldots, \tilde{x}_N^T]^T$, $\tilde{z} = [\tilde{z}_2^T, \tilde{z}_3^T, \ldots, \tilde{z}_N^T]^T$ with $\tilde{x}_k = [\tilde{x}_{k,1}, \tilde{x}_{k,2}, \ldots, \tilde{x}_{k,n}]^T$, $\tilde{z}_k = [\tilde{z}_{k,1}, \tilde{z}_{k,2}, \ldots, \tilde{z}_{k,n}]^T$, $k = 2, 3, \ldots, N$.

The constants c_1 can be chosen such that $c_1 \leq \frac{\beta_5}{2n}$, and we can get

$$\|\tilde{x} - \tilde{z}\|^2 + \|\tilde{z}\|^2 \leq \frac{1}{\mu_1}\gamma^{-\beta_7} e^{\delta\beta_6\frac{1}{c_1}(1-\frac{1}{\gamma})} V(0),$$

which indicates that

$$\lim_{t\to+\infty} \|\tilde{x}_k\| = 0, \quad \lim_{t\to+\infty} \|\tilde{z}_k\| = 0, \quad k = 2, 3, \ldots, N.$$

Thus, under the protocols (20) and (15), the nonlinear multi-agent system (1) can achieve the leader–follower regulation. ■

4. A numerical example

To illustrate the designed protocols, we consider a group of 5 agents with identical single-input single-output nonlinear dynamics, which is indexed by 1,2,3,4,5. In this multi-agent system, the agent labelled by 1 is referred as the leader and the agents labelled by 2,3,4,5 are called the followers. We also assume that all the follower agents know the input of the leader agent. For $k = 1, 2, 3, 4, 5$,

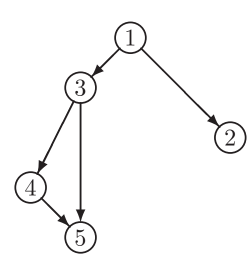

Figure 1. Communication topology.

agent k has the following identical feedforward dynamic:

$$\begin{aligned}\dot{x}_{k,1} &= x_{k,2} + \delta(t)\tanh(x_{k,3}) + \ln(1+\delta^2(t)u_k),\\ \dot{x}_{k,2} &= x_{k,3} + \delta(t)\tanh(u_k),\\ \dot{x}_{k,3} &= u_k,\\ y_k &= x_{k,1},\end{aligned} \quad (28)$$

where $x_k = [x_{k,1}, x_{k,2}, x_{k,3}]^T \in \mathbb{R}^3$ is the state of agent k, $u_k \in \mathbb{R}$ and $y_k \in \mathbb{R}$ are the input and output of agent k, respectively. $\delta(t)$ is an unknown bounded function whose boundedness is unknown a priori.

The communication topology graph is shown in Figure 1.

Then, the information weight matrix $\mathcal{A}$ and the Laplacian matrix $\mathcal{L}$ can be expressed as

$$\mathcal{A} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 1 & 0 \end{pmatrix},$$

$$\mathcal{L} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0\\ -1 & 1 & 0 & 0 & 0\\ -1 & 0 & 1 & 0 & 0\\ 0 & 0 & -1 & 1 & 0\\ 0 & 0 & -1 & -1 & 2 \end{pmatrix}.$$

We can achieve that this multi-agent system satisfies Assumption 2.1 and the network graph contains a directed spanning tree. With $\eta = 1$, the definition

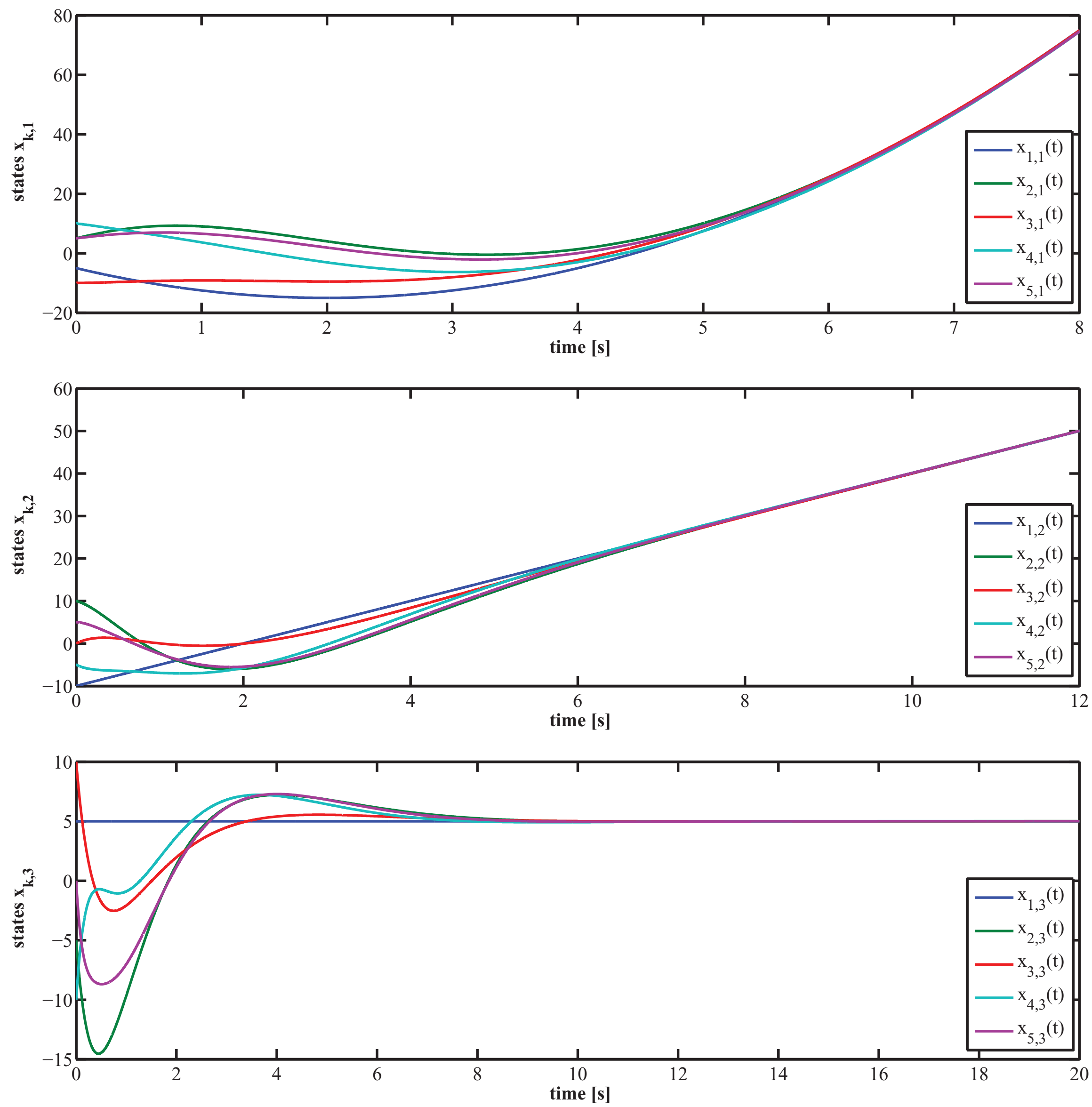

Figure 2. The states response of the closed-loop systems (28) and (29).

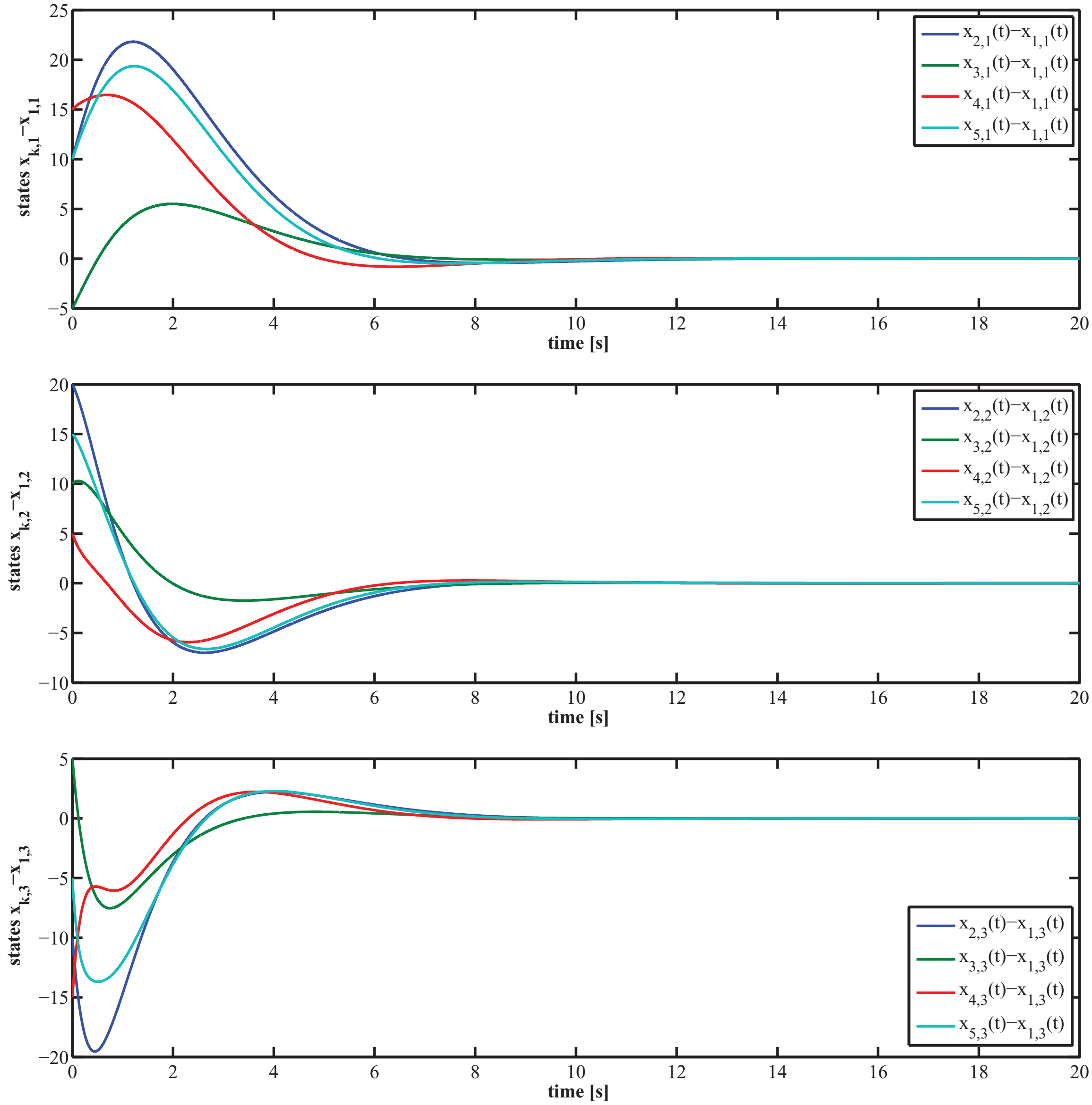

Figure 3. The errors response of the closed-loop systems (28) and (29).

of P, R can be expressed as follows by employing both Lemma 2.2 and Lemma 2.3,

$$P = \begin{pmatrix} 2.4142 & 2.4142 & 1.0000 \\ 2.4142 & 4.8284 & 2.4142 \\ 1.0000 & 2.4142 & 2.4142 \end{pmatrix},$$

$$R = \begin{pmatrix} 1.2 & 0 & 0 & 0 \\ 0 & 1.6 & 0 & 0 \\ 0 & 0 & 1.1 & 0 \\ 0 & 0 & 0 & 0.5 \end{pmatrix}.$$

When the initial condition of the agent state is different, for example, $x_1 = [-5, -10, 5]^T$, $x_2 = [5, 10, -5]^T$, $x_3 = [-10, 0, 10]^T$, $x_4 = [10, -5, -10]^T$, $x_5 = [5, 5, 0]^T$, the regulation protocol is necessary to be constructed. To render the multi-agent system leader–follower regulation, we present both the state feedback protocol and the output feedback protocol based on the designed procedure before.

For the state feedback case, the regulation form can be designed as

$$u_k = \alpha B^T P \sum_{j=1}^{5} \Gamma a_{kj}(x_j - x_k) + u_1, \; k = 2, 3, 4, 5, \tag{29}$$

where $B = [0, 0, 1]^T$, $\Gamma = \mathrm{diag}\{\frac{1}{\gamma^3}, \frac{1}{\gamma^2}, \frac{1}{\gamma^1}\}$ with γ being determined later, α is to be determined, and a_{ij} is the entry of $\mathcal{A}$.

From the designed procedure in Section 3.1, we can choose $\alpha = 1.6$, $\gamma = 0.02t + 1$. Figure 2 shows the simulation results under the condition $u_1(t) = 0$. The state errors are presented in Figure 3. It is obvious that multi-agents system (28) with protocol (29) can achieve leader–follower regulation.

For the output feedback case, the regulation form can be designed as:

$$u_k = \alpha B^T P \sum_{j=1}^{5} \Gamma a_{kj}(z_j - z_k) + u_1, k = 2, 3, 4, 5, \tag{30}$$

where $B = [0, 0, 1]^T$, and $z_k = [z_{k,1}, z_{k,2}, z_{k,3}]^T$ is the state of the following system:

$$\begin{aligned} \dot{z}_{k,1} &= z_{k,2} + 1.1\gamma^{-1}(y_k - z_{k,1}), \\ \dot{z}_{k,2} &= z_{k,3} + 0.8\gamma^{-2}(y_k - z_{k,1}), \\ \dot{z}_{k,3} &= u_k + 0.1\gamma^{-3}(y_k - z_{k,1}). \end{aligned} \tag{31}$$

Let the initial condition of the observer system as $z_k(0) = 0$, $k = 1, 2, 3, 4, 5$. Moreover, $\alpha = 1.6$ and $\gamma = 0.005t + 1$ are determined from the designed procedure in Section 3.1. Therefore, the simulation results are shown in Figure 4 when $u_1(t) = 0$. The error responses is shown in Figure 5. It can be found that the protocols (30)

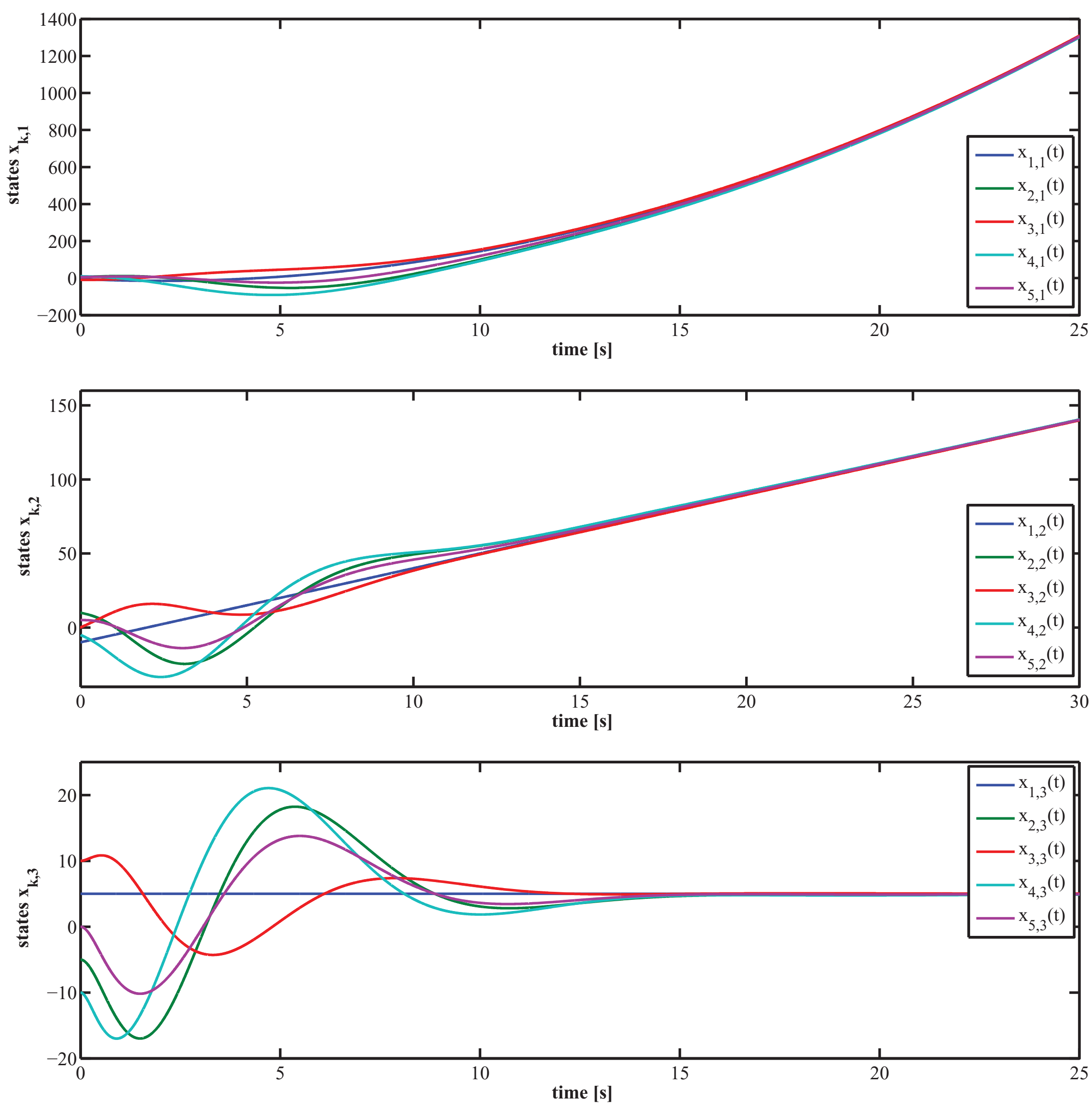

Figure 4. The states response of the closed-loop systems (28), (30) and (31).

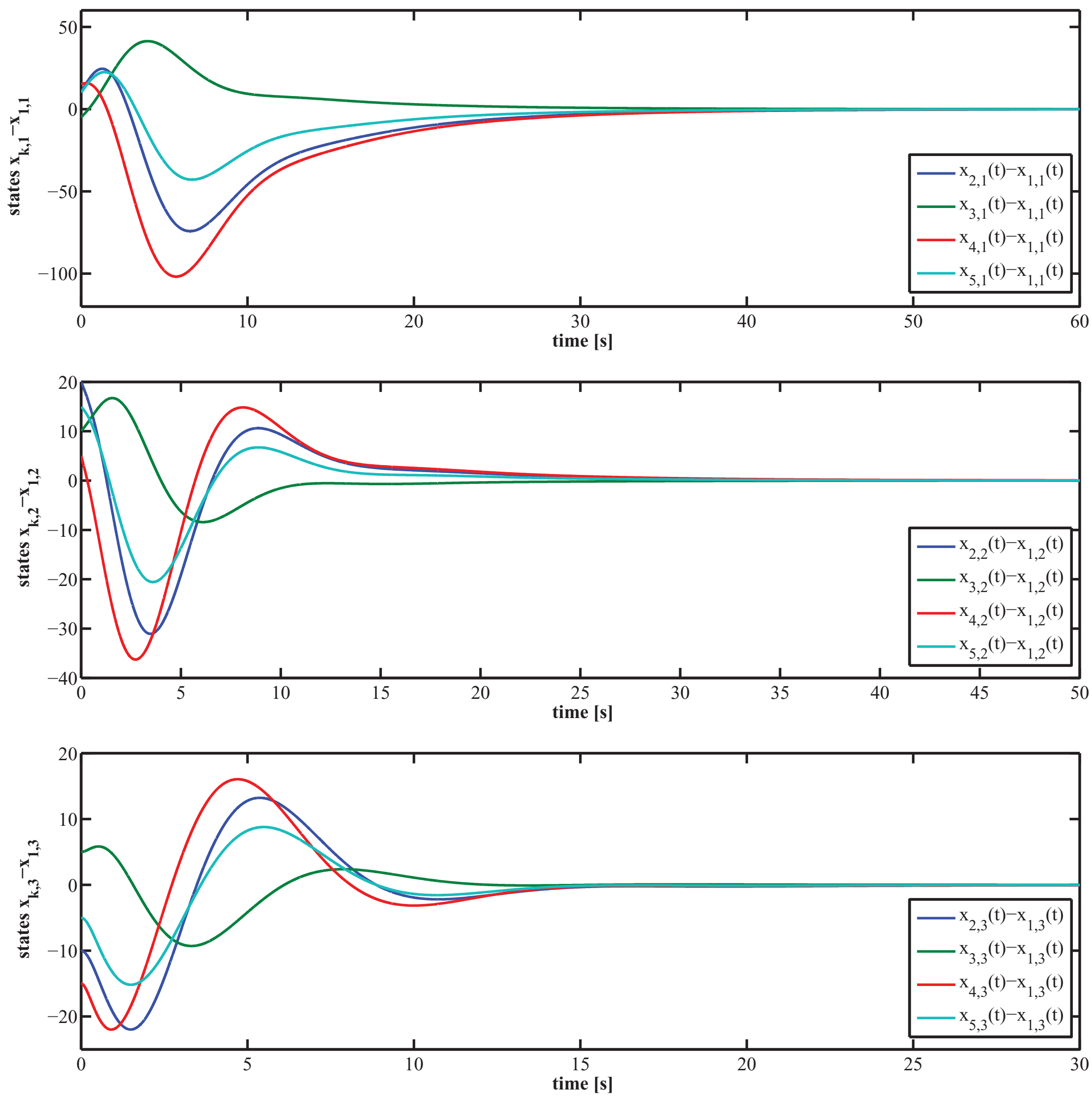

Figure 5. The errors response of the closed-loop systems (28), (30) and (31).

and (31) can render the multi-agent system (28) leader–follower regulation.

5. Conclusion

This paper shows that the genuine regulation algorithms, which takes full advantage of neighbour states and outputs, are effective to render the uncertain leader–follower nonlinear multi-agent systems regulation for all initial state values. A decentralised dynamic low-gain technology is introduced, where the rate of the dynamic gain is determined to dominate the unknown feedforward systems. An example, which has obvious nonlinear characteristics, has been presented to show that the performance could be achieved under the proposed regulation algorithms. One possible future topic is to consider heterogeneous nonlinear multi-agent systems with each agent described by different uncertain feedforward nonlinear dynamics.

Acknowledgment

The authors wish to thank the anonymous referees and the editor for their constructive comments and suggestions, which improved the presentation of this paper.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work is supported by the Major International (Regional) Joint Research Project of the National Natural Science Foundation of China (NSFC) [grant number 61320106011]; The national high technology research and development program (863Program) [grant number 2014AA052802]; The National Natural Science Foundation of China [grant number 61503214, 61573215 and 61573224].

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