On Designing Distributed Prescribed Finite-Time Observers for Strict-Feedback Nonlinear Systems

Le Chang, Qing-Long Han, *Fellow, IEEE*, Xiaohua Ge, *Member, IEEE*, Chenghui Zhang, *Senior Member, IEEE*, and Xianfu Zhang, *Member, IEEE*

***Abstract*—This article is concerned with the problem of distributed prescribed finite-time observer design for a strict-feedback nonlinear system with external disturbance. The purpose is to reconstruct the unavailable system state based on a group of distributed observers, where each of them can only receive at most 1-D output measurement from the system. First, in the absence of disturbance, a new distributed prescribed finite-time observer featuring time-varying gains is constructed and designed under the assumption of joint observability. It is analytically proved that for any prescribed instant independent of system initial conditions and other design parameters, the obtained distributed observer can guarantee not only the asymptotic convergence to zero of the state error between each observer state and the system state at this prescribed instant but also the definite zero-state error after this prescribed instant. Second, a distributed prescribed finite-time bounded observer is delicately proposed to account for the presence of external disturbance in the system dynamics. It is shown that the state error can be bounded by an arbitrarily positive constant after a prescribed instant. Finally, a numerical example and an electromechanical system are presented to demonstrate the effectiveness of the proposed results.**

***Index Terms*—Distributed observers, prescribed finite-time boundedness, prescribed finite-time convergence, strict-feedback nonlinear systems.**

I. Introduction

During the past decade, the design of the distributed observer has attracted extensive attention in response to an increasing demand to estimate the unavailable state of a dynamical system over spatially deployed sensors (or agents) due to either limited sensing capability, high-order system dynamics, high cost of a direct measurement method, or the existence of external disturbance and measurement noise (see [1]–[11] and references therein). The aim of distributed observer design is to reconstruct the unavailable system state through a network of interacting observers. The main design challenge lies in that the measurement output received by each observer may not be able to reconstruct the full/complete state of the system due to the fact that sensors have insufficient sensing capability or suffer from missing measurement (or data losses) through the wireless data-transmission channels. Until now, several distributed observer design methods have been proposed in the literature. To name a few, in [1], each observer agent fully converged to the system at any pro-given rate by properly choosing the gain matrices. These gain matrices involved the constant state transformations which divided each observer agent dynamics into an observable part and an unobservable part. In [2], a condition based on a linear matrix inequality was derived to determine the matrix gain of the proposed distributed observer for a time-invariant linear system. In [3] and [4], the robust distributed estimation was studied, where the error state between the system state and each observer state converged to zero in a finite time without perturbation. Nevertheless, most of the existing results on distributed observer design are limited to linear time-invariant systems.

It is well acknowledged that nonlinearities are not uncommon in practical systems, such as mechanical connections, hydraulic actuators, electric servomotors, and other application systems. The strict-feedback nonlinear system, also called the lower-triangular nonlinear system, represents an important category of nonlinear systems. The structure of a strict-feedback nonlinear system often includes a linear part and a nonlinear part possessing strict nonlinear couplings. Note that when the coefficients of the linear part are nonzero, several methods, such as the backstepping method and the high gain control method, have been available to deal with these nonlinear couplings. (See [12]–[21] on observer design, [22] on tracking control design, and [23]–[29] on consensus protocol design.) Whereas, for other practical systems, such as an electromechanical system in a mutual inductance transformer circuit, some of the coefficients of the linear part can be zero. In this sense, the 1-D measurement output is not sufficient to reconstruct all or even parts of the system state components for the strict-feedback nonlinear system. Such an issue is ineluctable when a network of observers is involved and each observer is capable of collecting at most the 1-D system state information from the strict-feedback nonlinear system. In this case, how to achieve the design of a network of interacting distributed observers such that each observer is able to reconstruct the

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L. Chang, C. Zhang, and X. Zhang are with the School of Control Science and Engineering, Shandong University, Jinan 250061, China (e-mail: le@lchang.me; zchui@sdu.edu.cn; zhangxianfu@sdu.edu.cn).

Q.-L. Han and X. Ge are with the School of Software and Electrical Engineering, Swinburne University of Technology, Melbourne, VIC 3122, Australia (e-mail: qhan@swin.edu.au; xge@swin.edu.au).

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state for a strict-feedback nonlinear system serves as the first motivation of this article.

The settling time, which can be adopted to characterize the convergence rate of the state error between an observer state and the system state, in the observer design literature, is deemed as an important performance indicator for evaluating the effectiveness of a finite-time observer design method. There is no doubt that fast convergence is desired in practice so as to achieve better system performance and robustness [30]. However, most existing distributed observer design methods are only capable of estimating/reconstructing the system's state either at an asymptotic convergence rate [1] or in a finite-time convergence [3]. Moreover, the settling time with finite-time performance often relies on the system initial conditions and, thus, is generally difficult to verify because the system initial conditions may be unavailable or inaccurate or private in some practical scenarios. To address this concern, fixed-time performance offers an alternative in the sense that an estimate or upper bound of the settling time can be derived without dependence on the system initial states [31]. Nevertheless, it should be also pointed out that it is mathematically complicated to derive an estimate or upper bound for the settling time when designing such a fixed-time converging distributed observer. Moreover, the estimated or bounded settling time still represents a conservative solution because it cannot be preassigned arbitrarily and may be larger than the actual convergence time. To the best of our knowledge, how to design distributed observers for a strict-feedback nonlinear system *in an explicitly prescribed finite convergence time* is much more challenging and remains open, which is the second motivation of this article.

Practical systems are often affected by unexpected system component failures or unknown exogenous inputs which can be deemed as external disturbance acting on the system dynamics [32]–[37]. Undeniably, such an external disturbance signal will pose significant difficulties on the system performance evaluation and distributed finite-time observer design beyond the prescribed finite time. Therefore, how to deal with the distributed observer design in an explicitly prescribed finite convergence time *when disturbance inevitably occurs in the system dynamics* is the third motivation of this article.

In this article, we will address the design of distributed prescribed finite-time observers for a strict-feedback nonlinear system with/without disturbance. The main contributions are summarized as follows.

1) *Partially available sensor measurements* will be incorporated for observer design. Different from the classical observer design problem or the observer-based multiagent consensus-seeking problem, each of the proposed distributed observers will not require the system state to be completely observable for facilitating the design procedure.
2) *A general system model of the nonlinear strict-feedback form under disturbance* will be considered. The concerned nonlinear system is over a prescribed finite-time interval rather than the whole time horizon. Moreover, the nonlinear terms are assumed to satisfy the Lipschitz condition with an unknown Lipschitz constant on this finite-time interval. Such a requirement will accommodate more general nonlinear characterizations, such as the nonlinear terms satisfying the constant Lipschitz condition or the time-varying condition or even the state-dependent condition. Thus, new observer design methods are imperative for such a nonlinear system.
3) *Refined concepts of distributed prescribed finite-time convergence and distributed prescribed finite-time boundedness* will be proposed. It will be shown that the state of each observer will be equal to that of the system after a prescribed instant. The proposed distributed prescribed finite-time boundedness, on the other hand, will guarantee a bounded state error under any positive constant after this time instant.
4) *New methods for designing desired distributed prescribed finite-time observer* will be developed. When disturbance is absent, the designed observer can ensure that for an arbitrarily chosen prescribed time $T^* \in (0, T)$ not only does the observer state converge to the system state on the time interval $[0, T^*)$ but also the state error becomes zero on $[T^*, T)$. When the disturbance is present in system dynamics and bounded, by employing a switching observer gain, the designed observer can guarantee a bounded state error under any positive constant on $[T^*, T)$.

The remainder of this article is organized as follows. Section II recalls some basic concepts of graph theory, formulates the problem, and presents some useful lemmas. Section III states the main theoretical results on designing the desired distributed prescribed finite-time observer and distributed prescribed finite-time bounded observer. Section IV verifies the main results through a numerical example and an electromechanical system. Section V gives some concluding remarks.

Notation: Denote by $\mathbb{R}$ the field of real numbers, $\mathbb{R}^m$ the set of m real column vectors, and $\mathbb{R}^{m\times n}$ the set of $m \times n$ real matrices. $\mathbf{1}_N$ stands for the N-dimensional column vector with each element being 1. I_n represents the identity matrix of size n. 0 is employed to denote the matrix with all elements being zero.

II. Preliminaries and Problem Formulation

A. Graph Theory

Some basic concepts of algebraic graph theory are recalled. Consider a directed connected graph $\mathcal{G}(\mathcal{N}, \mathcal{E}, \mathcal{A})$, where $\mathcal{N} = \{1, 2, \ldots, N\}$ is a set of N nodes, $\mathcal{E} \in \mathcal{N} \times \mathcal{N}$ is an edge set of paired nodes, and $\mathcal{A} = (a_{ij}) \in \mathbb{R}^{N\times N}$ is the adjacency matrix. When there is an information link from the jth node to the ith node, one has $(i, j) \in \mathcal{E}$ and $a_{ij} > 0$; otherwise, $(i, j) \notin \mathcal{E}$ and $a_{ij} = 0$. It is assumed that no self-loop exists, that is, $a_{ii} = 0$. The element of the Laplacian matrix $\mathcal{L} = (l_{ij}) \in \mathbb{R}^{N\times N}$ of graph $\mathcal{G}$ is defined as $l_{ij} = -a_{ij}, i \neq j$ and $l_{ii} = \sum_{j=1}^{N} a_{ij}$. Apparently, zero is one of the eigenvalues of $\mathcal{L}$ and $\mathcal{L}\mathbf{1}_N = 0$. A directed path from node i_1 to i_l is an ordered sequence of edges $(i_k, i_{k+1}), k = 1, 2, \ldots, l-1$ in the graph. A directed graph $\mathcal{G}$ is strongly connected if there exists a directed path from node i to node j between any pair of distinct nodes i and j in $\mathcal{G}$. To facilitate subsequent analysis, two existing lemmas on the strongly connected graph are recalled.

Lemma 1 [38]: Suppose that $\mathcal{L} = (l_{ij}) \in \mathbb{R}^{N\times N}$ denotes the Laplacian matrix of a strongly connected digraph $\mathcal{G}$. There exists a unique vector $\xi \triangleq (\xi_1, \xi_2, \ldots, \xi_N)^T \in \mathbb{R}^N$ satisfying $\xi_i > 0$, $\forall\ i = 1, 2 \ldots, N$ such that $\xi^T\mathcal{L} = 0$ and $\xi^T\mathbf{1}_N = N$. Denote $\tilde{\mathcal{L}} \triangleq \Xi\mathcal{L} + \mathcal{L}^T\Xi$, where $\Xi \triangleq \mathrm{diag}\{\xi_1, \xi_2, \ldots, \xi_N\}$. The eigenvalues of $\tilde{\mathcal{L}}$ can be arranged as $0 = \lambda_1 < \lambda_2 \le \cdots \le \lambda_N$. Moreover, there exists an orthogonal matrix $U = ([1/\sqrt{N}]\mathbf{1}_N \quad U_2)$, where $U_2 \in \mathbb{R}^{N\times(N-1)}$ such that $U^T\tilde{\mathcal{L}}U = \mathrm{diag}\{0, \lambda_2, \ldots, \lambda_N\}$.

Lemma 2: Suppose that $\tilde{\mathcal{L}}$ is defined in Lemma 1. Then, one has

$$\tilde{\mathcal{L}} \ge \lambda_2 I_N - \lambda_2\frac{1}{N}\mathbf{1}_N\mathbf{1}_N^T. \tag{1}$$

Proof: From Lemma 1, we have that $U^T\tilde{\mathcal{L}}U = \mathrm{diag}\{0, \lambda_2, \ldots, \lambda_N\}$. Then, $\tilde{\mathcal{L}} = U\mathrm{diag}\{\lambda_2, \lambda_2, \ldots, \lambda_N\}U^T - \lambda_2 U\mathrm{diag}\{1, 0, \ldots, 0\}U^T \ge \lambda_2 I_N - \lambda_2(1/N)\mathbf{1}_N\mathbf{1}_N^T$, where $\lambda_2 \le \cdots \le \lambda_N$ is utilized. ■

Remark 1: It should be noted that when $\mathbf{1}_N^T x = 0$ for some $x \in \mathbb{R}^N$, inequality (1) implies that $x^T\tilde{\mathcal{L}}x \ge \lambda_2 x^T x$, which has been widely employed to deal with consensus-seeking problems (see [38], [39]).

B. Model of Strict-Feedback Nonlinear System Under Disturbance

The plant to be observed is modeled by a strict-feedback nonlinear system under external disturbance of the following continuous-time state-space form:

$$\dot{X} = AX + F(t, X) + Bd(t) \tag{2}$$

where $X = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$ is the state of the system, $d(t) : [0, T) \to \mathbb{R}^n$ denotes the continuous and bounded disturbance input, $B = (0\ 0\ \cdots\ 0\ 1)^T \in \mathbb{R}^{n\times 1}$, $A \in \mathbb{R}^{n\times n}$, and the nonlinear term $F(t, X) : [0, T) \times \mathbb{R}^n \to \mathbb{R}^n$ have the following forms:

$$A = \begin{pmatrix} 0 & \delta_1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \delta_{n-1} \\ 0 & 0 & \cdots & 0 \end{pmatrix}$$

$$F(t, X) = \begin{pmatrix} f_1(t, x_1) \\ f_2(t, x_1, x_2) \\ \vdots \\ f_n(t, x_1, x_2, \ldots, x_n) \end{pmatrix}$$

with $\delta_i, i = 1, 2, \ldots, n-1$ being 0 or 1, and $F(t, 0) = 0$.

Assumption 1: For any $x_k, z_k \in \mathbb{R}$, $k = 1, 2, \ldots, j, j = 1, 2, \ldots, n$, when $t \in [0, T)$, the following inequality:

$$\begin{aligned} &\left|f_j\big(t, x_1, x_2, \ldots, x_j\big) - f_j\big(t, z_1, z_2, \ldots, z_j\big)\right| \\ &\quad \le \theta\big(|x_1 - z_1| + |x_2 - z_2| + \cdots + |x_j - z_j|\big) \end{aligned}$$

holds, where θ is an unknown constant.

Remark 2: The nonlinearities in the existing study of the strict-feedback systems are often required to meet a time-varying Lipschitz condition that $|f(t, x) - f(t, y)| \le e^t|x - y|$, $t \in [0, +\infty)$ (see [12], [13], [16]–[18], [29]) or a state-dependent Lipschitz condition that $|f(t, x) - f_i(t, y)| \le c(x)|x - y|$, $t \in [0, +\infty)$ with $c(x) : \mathbb{R}^n \to \mathbb{R}^+$ being a continuous function (see [19]–[21]). In practice, the nonlinear system dynamics may be perturbed largely over a long timescale due to the fact that structural material degradation occurs in a long-term operation. In this case, the existing convergence evaluation methods, including the asymptotic convergence evaluation method or the finite-time convergence evaluation method, may be invalid. Assumption 1 indicates that the nonlinear terms $f_i(\cdot)$ satisfy the Lipschitz condition for $t \in [0, T)$, which guarantees that system (2) under any initial condition has a unique solution. Moreover, the time horizon under consideration is $[0, T)$ rather than $[0, +\infty)$, which accommodates more general nonlinear characterizations. For example, the nonlinear terms obeying the constant Lipschitz condition or the time-varying condition or even the state-dependent condition can meet Assumption 1 on the finite-time interval $[0, T)$.

C. Model of Sensor Measurement Output

N distributed sensors are spatially deployed to monitor the plant while only m ($m \le N$) sensors can actually access the partial state information of the system, as shown in Fig. 1. Moreover, due to the insufficient sensing capability and limited battery, each of these m sensors may be capable of measuring only the 1-D state component in practice. In this sense, the system measurement output model for each of these m sensors can be described by

$$y_i = x_{p_i} \in \mathbb{R}, \quad i = 1, 2, \ldots, m, \quad p_i \in \{1, 2, \ldots, n\} \tag{3}$$

or can be given in the matrix form of

$$y_i = C_i X, \quad i = 1, 2, \ldots, m \tag{4}$$

with $C_i = (0 \quad \ldots \quad 0 \quad \underbrace{1}_{\text{The } p_i\text{th element}} \quad 0 \quad \ldots \quad 0) \in \mathbb{R}^{1\times n}$. For those sensors, indexed by $m+1, m+2, \ldots, N$, which cannot directly measure the system state X, we simply represent the sensor measurement output model as

$$y_i = C_i X = 0, \quad i = m+1, m+2, \ldots, N \tag{5}$$

by noting that $C_i = 0$. Denote the joint matrix C as follows:

$$C = \left(C_1^T \ \ldots \ C_m^T\right)^T \in \mathbb{R}^{m\times n}. \tag{6}$$

D. Structure of the Proposed Distributed Prescribed Finite-Time Observers

We are interested in constructing and designing a network of interacting distributed observers of the following form:

$$\begin{aligned} \dot{X}_i &= AX_i + F(t, X_i) - L_i(y_i - C_i X_i) \\ &\quad - K_i\sum_{k=1}^{N} a_{ik}(X_i - X_k), \quad i \in \mathcal{N} \end{aligned} \tag{7}$$

where $X_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,n})^T \in \mathbb{R}^n$ is the state of the ith observer, and L_i and K_i are the gain matrices to be determined later. The schematic of the proposed distributed observers for the considered system (2) is illustrated in Fig. 1.

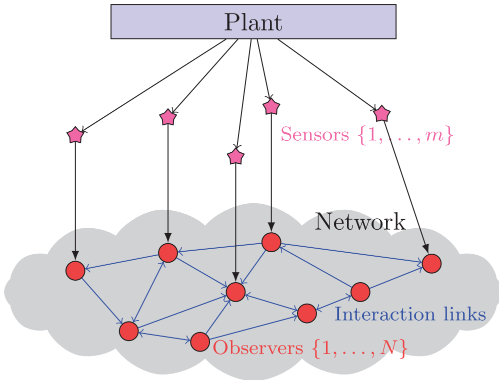

Fig. 1. Schematic of the proposed distributed observers.

Note that the above distributed observer structure has been widely adopted in the existing literature. (See [1]–[4] and the references therein.) However, the system dynamics were restricted to be linear and only the asymptotical convergence performance was considered (see [1], [2]). Although the finite-time convergence was addressed in [3] and [4], the proposed distributed finite-time observer only guaranteed the convergence of the state error in a finite time. In this article, we aim to guarantee the distributed prescribed finite-time convergence performance and the distributed prescribed finite-time boundedness convergence performance. In doing so, the following definitions are presented.

Definition 1: Suppose that system (2) without disturbance (i.e., $d(t) = 0$) is defined on the finite-time interval $[0, T)$. It is said that the state error $X(t) - X_i(t)$, $i \in \mathcal{N}$ on each observer achieves *distributed prescribed finite-time convergence* if for arbitrarily selected prescribed time $T^* \in (0, T)$, there exists a suitable distributed observer of the form (7) such that for any initial conditions $X(0), X_1(0), \dots, X_N(0) \in \mathbb{R}^n$, the following conditions hold:

$$\lim_{t \to T^*} \|X(t) - X_i(t)\| = 0 \tag{8}$$

$$\|X(t) - X_i(t)\| = 0, \quad t \in [T^*, T). \tag{9}$$

In this sense, observer (7) is called a *distributed prescribed finite-time observer*.

Definition 2: Suppose that system (2) with disturbance (i.e., $d(t) \neq 0$) is defined on the finite-time interval $[0, T)$. It is said that the state error $X(t) - X_i(t)$, $i \in \mathcal{N}$ on each observer achieves *distributed prescribed finite-time boundedness* if for arbitrarily selected prescribed time $T^* \in (0, T)$, there exists a suitable distributed observer of the form (7) such that

$$\|X(t) - X_i(t)\| \le T^* \vartheta, \quad t \in [T^*, T) \tag{10}$$

holds for any initial conditions $X(0), X_1(0), \dots, X_N(0) \in \mathbb{R}^n$, where ϑ is a positive constant. In this sense, observer (7) is called a *distributed prescribed finite-time bounded observer*.

Remark 3: It is noteworthy that a model of partially available sensor measurement outputs in the form of (3)–(5), namely, only $m(\le N)$ out of N sensors can measure the 1-D state information of the plant, is employed for the proposed distributed observer design. Furthermore, the observability for each pair of (C_i, A), $i \in \mathcal{N}$, as exposed in the classical observer design problem or the observer-based multiagent consensus-seeking problem, is no longer required. In this sense, each observer of the form (7) only knows *at most* the 1-D measurement output in terms of C_i. In other words, the observer gain L_i cannot be determined to ensure that all eigenvalues of matrix $(A + L_i C_i)$ for any $i \in \mathcal{N}$ are negative real numbers or complex numbers with negative real parts, which represents a necessity for the traditional Luenberger observer design methods. Thus, the techniques for achieving the design of the traditional Luenberger observers cannot be directly employed in the proposed distributed observer framework.

Remark 4: From Definition 1, one can clearly see that the proposed distributed prescribed finite-time observer not only ensures the state convergence on a prescribed finite-time interval $[0, T^*)$ but also requires the state of each observer to be equal to that of the system on the time interval $[T^*, T)$. In the following section, it will be shown that the prescribed convergence time T^* does not rely on the initial condition but can be arbitrarily chosen rather than estimated or bounded, which represents a distinct feature of the proposed observer compared with some existing ones [30], [40]. Although the prescribed finite-time convergence performance was introduced in [41] to study the consensus for multiagent systems, we focus on the observer design and its robustness performance.

E. Problem to Be Addressed

The *distributed prescribed finite-time observer design problem* to be addressed can be stated as follows. For the nonlinear system (2) confined on the finite-time interval $[0, T)$, given partially available sensor measurements y_i in the form of (4) and completely unavailable sensor measurements in the form of (5), the objective is to design gain matrices L_i and K_i, $i \in \mathcal{N}$ such that the distributed observer (7) is: 1) a distributed prescribed finite-time observer in the absence of disturbance $d(t)$ and 2) a distributed prescribed finite-time bounded observer in the presence of disturbance $d(t)$.

Before ending this section, the following lemmas, which facilitate the establishment of our main results, are provided.

Lemma 3: The mapping

$$\vartheta : p_i \in \{1, 2, \dots, n\} \to \tau_i \in \{1, 2, \dots, n\} \tag{11}$$

can be well defined in the sense that for given constants $\delta_1, \delta_2, \dots, \delta_{n-1}$ in matrix A, if $\delta_{p_i} = 0$, then $\tau_i = 1$; else τ_i is a minimum constant satisfying $\delta_{p_i + \tau_i - 1} = 0$ with $\delta_n = 0$. When the matrix pair (C, A) is observable, where C is defined in (6), one has $\{1, 2, \dots, n\} \subset \cup_{i=1,\dots,m}\{p_i, p_i + 1, \dots, p_i + \tau_i - 1\}$.

Proof: See Appendix A. ■

Lemma 4: Suppose that $a_1, a_2, \dots, a_N \in \mathbb{R}$ with $N \ge 2$. For any non-negative constant q_1, there exists a constant $q_2 \in [0, 1)$ such that

$$(a_1 + a_2 + \cdots + a_N)^2 \le q_1 a_1^2 + N(1 - q_2)(a_1^2 + a_2^2 + \cdots + a_N^2).$$

Furthermore, $q_2 = 0$ is satisfied only when $q_1 = 0$.

Proof: See Appendix B. ■

Remark 5: When $q_1 = 0$, one can obtain

$$(a_1 + a_2 + \cdots + a_N)^2 \le N\left(a_1^2 + a_2^2 + \cdots + a_N^2\right)$$

which is the general Cauchy–Schwarz inequality.

Lemma 5: Suppose that the matrices $A, C_i, i = 1, 2, \ldots, m$, are defined in (2) and (4), $H = \text{diag}\{n, n-1, \ldots, 1\}$, and

$$M_{\tau_i} = \text{diag}\left\{\begin{matrix} 0 & \cdots & 0 & \underbrace{1 \quad \cdots \quad 1} & 0 & \cdots & 0 \\ & & & \text{From } p_i \text{ element to } p_i + \tau_i - 1 \text{ element} & & & \end{matrix}\right\}.$$

For any constant γ, there exist a vector $\bar{L}_i$ and a positive-defined matrix P_i such that

$$\left(A + \bar{L}_i C_i\right)P_i + P_i\left(A + \bar{L}_i C_i\right) \le A + A^T - \gamma M_{\tau_i} \quad (12)$$

$$\alpha I_n \le P_i H + H P_i \le \beta I_n \quad (13)$$

where α and β are the positive constants.

Proof: See Appendix C. ■

III. Main Results

In this section, we first provide the design criterion for the desired distributed observer (7) in terms of $K_i, L_i, i \in \mathcal{N}$ such that the prescribed finite-time convergence performance on $[0, T)$ is preserved. Then, we consider a strict-feedback nonlinear system subject to disturbance $d(t)$ and design a suitable distributed observer in the form of (7) to guarantee the distributed prescribed finite-time boundedness.

A. Design of Distributed Prescribed Finite-Time Observer

Theorem 1: Under Assumption 1, consider system (2) without $d(t)$ over a network of N cooperative observers via a strongly connected directed network topology characterized by $\mathcal{G}(\mathcal{N}, \mathcal{E}, \mathcal{A})$. If the matrix pair (C, A) is observable, then for an arbitrarily chosen prescribed time $T^* \in (0, T)$, a distributed prescribed finite-time observer (7) exists with the observer gain matrices $K_i, L_i, i \in \mathcal{N}$, given as

$$\begin{cases} L_i = \frac{(T^*-t)^{-n-2+p_i}}{\mu^{-n-2+p_i}}\Phi^{-1}\bar{L}_i \\ K_i = \gamma\frac{\mu}{T^*-t}\Phi^{-1}P_i^{-1}\Phi \end{cases} \quad t \in \left[0, T^*\right)$$
$$\begin{cases} L_i = 0 \\ K_i = 0 \end{cases} \quad t \in \left[T^*, T\right) \quad (14)$$

where $\Phi = \text{diag}\{(\mu^n/[(T^* - t)^n]), (\mu^{n-1}/[(T^* - t)^{n-1}]), \ldots, (\mu/[T^* - t])\}$ with μ and γ being the positive constants, $\bar{L}_i, P_i, i = 1, 2, \ldots, m$ are defined in Lemma 5, and $\bar{L}_i = 0, P_i = I_n, i = m+1, m+2, \ldots, N$.

Proof: See Appendix D. ■

Based on Theorem 1, the following algorithm, namely Algorithm 1, which outlines the design procedure of the distributed prescribed finite-time observer (7) in terms of its gain matrices K_i and L_i, $i \in \mathcal{N}$, is presented.

Remark 6: As noted in the preceding section, the proposed distributed observer design method is different from some existing methods for designing classical observers or observer-based consensus control protocols in the multiagent literature. More specifically, in a classical observer, the gain matrix L can be designed such that the matrix $A + LC$ is Hurwitz so that the nonlinear terms can be tackled. Nevertheless, in the design of the distributed observer (7), the matrix pair (C_i, A) may be unobservable and the gain matrix L_i may not exist to ensure all eigenvalues of $A + L_i C_i$ to be negative. In this case, one needs to deal with constraint (22) in terms of a negative term and a positive term. On the other hand, in the design of a consensus protocol, given that the state error stratifies $\mathbf{1}_N^T x = 0$, the consensus term $K_i \sum_{k=1}^N a_{ik}(X_i - X_k)$ can be employed to render the state error negative whereas one fails to preserve $\mathbf{1}_N^T x = 0$ for the distributed observer (7), which inevitably leads to constraint (24) with regard to a negative term and a positive term. From the proposed design procedure, it can be seen that by properly selecting the coupling parameter γ for (22) and (24), the proposed distributed observer (7) can effectively deal with the nonlinear terms.

Algorithm 1 Distributed Prescribed Finite-Time Observer Design Procedure

Step 1.

Calculate $q_1 = \frac{N}{\lambda_2}\bar{\xi}$ where $\bar{\xi} = \min\{\xi_1, \xi_2, \ldots, \xi_N\}$ with $\xi = (\xi_1, \xi_2, \ldots, \xi_N)^T$ such that $\xi^T\mathcal{L} = 0$ and $\xi^T\mathbf{1}_N = N$ with $\mathcal{L}$ being the Laplacian matrix, where λ_2 denotes the minimum nonzero eigenvalue of matrix $\Xi\mathcal{L} + \mathcal{L}^T\Xi$ with $\Xi \triangleq \text{diag}\{\xi_1, \xi_2, \ldots, \xi_N\}$.

Step 2.

Find $\varepsilon = q_2\lambda_2$ such that q_2 satisfies the following inequality

$$(a_1+a_2+\cdots+a_N)^2 \le q_1 a_1^2 + N(1-q_2)(a_1^2+a_2^2+\cdots+a_N^2)$$

for any $a_1, a_2, \ldots, a_N \in \mathbb{R}$.

Step 3.

Solve out $\bar{L}_i, P_i, i = 1, 2, \ldots, m$ such that

$$\left(A + \bar{L}_i C_i\right)P_i + P_i\left(A + \bar{L}_i C_i\right) \le A + A^T - \gamma M_{\tau_i}$$
$$P_i H + H P_i > 0$$

hold, where $\gamma = \frac{2\|A\|+1}{\varepsilon}$ for $H = \text{diag}\{n, n-1, \ldots, 1\}$, and

$$M_{\tau_i} = \text{diag}\{0 \quad \cdots \quad 0 \quad \underbrace{1 \quad \cdots \quad 1}_{\text{From } p_i \text{ element to } p_i + \tau_i - 1 \text{ element}} \quad 0 \quad \cdots \quad 0\}.$$

Step 4.

Select μ such that $\mu \ge \max\{\alpha N + 1, T^*\}$ with α being the minimum eigenvalue of matrix $P_i H + H P_i$.

Step 5.

Compute K_i and L_i, $i \in \mathcal{N}$ as follows

$$\begin{cases} L_i = \frac{(T^*-t)^{-n-2+p_i}}{\mu^{-n-2+p_i}}\Phi^{-1}\bar{L}_i, \\ K_i = \gamma\frac{\mu}{T^*-t}\Phi^{-1}P_i^{-1}\Phi, \end{cases} \quad t \in [0, T^*)$$
$$\begin{cases} L_i = 0, \\ K_i = 0, \end{cases} \quad t \in [T^*, T)$$

where $\Phi = \text{diag}\{\frac{\mu^n}{(T^*-t)^n}, \frac{\mu^{n-1}}{(T^*-t)^{n-1}}, \ldots, \frac{\mu}{T^*-t}\}$ and $\bar{L}_i = 0, P_i = I_n, i = m+1, \ldots, N$.

B. Design of Distributed Prescribed Finite-Time Bounded Observer

We are now in a position to state the following observer design criterion on guaranteeing the distributed prescribed finite-time boundedness for system (2) in the presence of external disturbance $d(t)$.

Theorem 2: Under Assumption 1, consider system (2) with $d(t)$ over a network of N cooperative observers via a strongly connected directed network topology characterized by $\mathcal{G}(\mathcal{N},\mathcal{E},\mathcal{A})$. If the matrix pair (C,A) is observable, then for any $T^* \in (0,T)$, there exists a distributed prescribed finite-time bounded observer (7) such that the state error $\eta(t) = (\eta_1^T, \eta_2^T, \ldots, \eta_N^T)^T$ with $\eta_i = X_i - X$

$$\|\eta(t)\| \leq T^*\Omega, \quad t \in [T^*, T) \tag{15}$$

holds for any initial conditions $X(0), X_1(0), \ldots, X_N(0) \in \mathbb{R}^n$, where Ω is a constant depending on T, θ, and the disturbance bound $\bar{d}$. Moreover, the observer gain matrices K_i and L_i, $i \in \mathcal{N}$, can be designed as

$$\begin{cases} L_i = \frac{(kT^*-t)^{-n-2+p_i}}{\mu^{-n-2+p_i}}\Phi^{-1}\bar{L}_i \\ K_i = \gamma\frac{\mu}{kT^*-t}\Phi^{-1}P_i^{-1}\Phi \end{cases} \tag{16}$$

for $t \in [(k-1)T^*, kT^*)$, $k = 1, 2, \ldots, \bar{k}$, and

$$\begin{cases} L_i = \frac{(T-t)^{-n-2+p_i}}{\mu^{-n-2+p_i}}\Phi^{-1}\bar{L}_i \\ K_i = \gamma\frac{\mu}{T-t}\Phi^{-1}P_i^{-1}\Phi \end{cases} \tag{17}$$

for $t \in [\bar{k}T^*, T)$, where $\Phi = \mathrm{diag}\{(\mu^n[(kT^*-t)^n]), (\mu^{n-1}/[(kT^*-t)^{n-1}]), \ldots, (\mu/kT^*-t)\}$ with μ, γ being positive constants; $\bar{L}_i, P_i, i = 1, 2, \ldots, m$ are defined in Lemma 5; $\bar{L}_i = 0, P_i = I_n, i = m+1, m+2, \ldots, N$; and $\bar{k}$ satisfies $\bar{k}T^* < T \leq (\bar{k}+1)T^*$.

Proof: See Appendix E. ■

Based on Theorem 2, one can similarly obtain an algorithm for determining the design of desired observer gain matrices K_i and L_i, $i \in \mathcal{N}$. For brevity, the design procedure is omitted.

Remark 7: From (15), one can observe that the bound of the state error $\eta(t)$ depends on the prescribed time T^* and an unknown constant Ω in terms of T, θ, and $\bar{d}$. Hence, in order to quantitatively analyze the effects of nonlinearity and disturbance on the evolution of state error, one can properly set a prescribed time T^* so as to gain a desired state error bound, which greatly increases the design freedom.

IV. Illustrative Examples

In this section, two illustrative examples are provided to verify the effectiveness of the proposed results.

A. Numerical Example

Suppose that $n = 4$, $m = 2$, $N = 5$, and $\delta_1 = \delta_3 = 1$, $\delta_2 = 0$ in matrix A. The nonlinear terms are chosen as $f_1(t,x_1) = 1.5\tan(t)x_1$, $f_2(t,x_1,x_2) = 1.5t^2x_1 + 1.5tx_2$, $f_3(t,x_1,x_2,x_3) = 1.5\cos(t)x_2 + 1.5\sin(t)x_3$, and $f_4(t,x_1,x_2,x_3,x_4) = 1.5tx_3 + 1.5t^2x_4$. It can be seen from the nonlinear term $f_1(t,x_1)$ that the system state x_1 tends to ∞ when t approaches $\pi/2$ s, which makes the observer design over the entire time domain

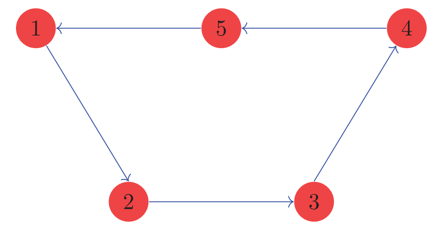

Fig. 2. Network topology of five interacting observers for Example IV-A.

$[0,+\infty)$ infeasible. In this example, under Assumption 1, we set $T = 1.5$ s and consider the convergence performance of the state error on the finite-time time interval $[0, 1.5)$.

The measurement output matrices are given as $C_1 = (1\ \ 0\ \ 0\ \ 0)$, $C_4 = (0\ \ 0\ \ 1\ \ 0)$, and $C_2 = C_3 = C_5 = 0$. The joint matrix C has the form

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

which meets the observability assumption of (C,A).

The network topology of five interacting observers is shown in Fig. 2, where the adjacency element $a_{ij} = 1$ if $(i,j) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. From Lemma 4, we choose $\varepsilon = 0.02$, thus yielding $\gamma = 191$. Employing Lemma 5, matrices $\bar{L}_1, \bar{L}_2, \bar{L}_3, \bar{L}_4, \bar{L}_5$ and P_1, P_2, P_3, P_4, P_5 can be selected as $\bar{L}_1 = (-2\ \ -1\ \ 0\ \ 0)^T$, $\bar{L}_4 = (0\ \ 0\ \ -2\ \ -1)^T$, $\bar{L}_2 = \bar{L}_3 = \bar{L}_5 = 0$, $P_2 = P_3 = P_5 = I_4$, and

$$P_1 = \begin{pmatrix} 21 & 21 & 0 & 0 \\ 21 & 64 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad P_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 21 & 21 \\ 0 & 0 & 21 & 64 \end{pmatrix}.$$

To achieve the design of the distributed prescribed finite-time observer, we choose a prescribed time as $T^* = 1$ s. Following (14), the observer gain matrices L_1, L_2, K_1, K_2, K_3 can be determined by letting $\Phi = \mathrm{diag}\{(\mu^4/[(1-t)^4]), (\mu^3/[(1-t)^3]), (\mu^2/[(1-t)^2]), (\mu/[1-t])\}$. Furthermore, we let $\mu = 11$ and achieve, for $t \in [0,1)$

$$L_1 = \begin{pmatrix} -\frac{22}{1-t} \\ -\frac{121}{(1-t)^2} \\ 0 \\ 0 \end{pmatrix}, \quad L_4 = \begin{pmatrix} 0 \\ 0 \\ -\frac{22}{1-t} \\ -\frac{121}{(1-t)^2} \end{pmatrix}$$

and

$$K_2 = \begin{pmatrix} -\frac{146.6}{1-t} & 4.5 & 0 & 0 \\ \frac{538.5}{(1-t)^2} & -\frac{49.0}{1-t} & 0 & 0 \\ 0 & 0 & -\frac{2101}{1-t} & 0 \\ 0 & 0 & 0 & -\frac{2101}{1-t} \end{pmatrix}$$

$$K_4 = \begin{pmatrix} -\frac{2101}{1-t} & 0 & 0 & 0 \\ 0 & -\frac{2101}{1-t} & 0 & 0 \\ 0 & 0 & -\frac{146.6}{1-t} & 4.5 \\ 0 & 0 & \frac{538.5}{(1-t)^2} & -\frac{49.0}{1-t} \end{pmatrix}$$

and $L_2 = L_3 = L_5 = 0$, $K_2 = K_3 = K_5 = (2101/[1-t])I_4$. When $t \in [1, 1.5)$, one has $L_1 = L_2 = L_3 = L_4 = L_5 = 0$ and $K_1 = K_2 = K_3 = K_4 = K_5 = 0$.

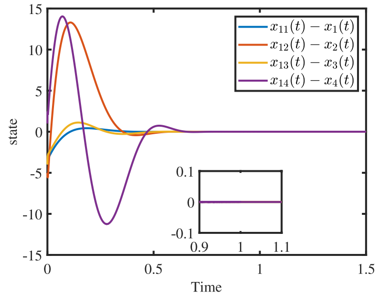

Fig. 3. Trajectory of state error $x_{1,j}-x_j$, $j=1,2,3,4$ on observer 1.

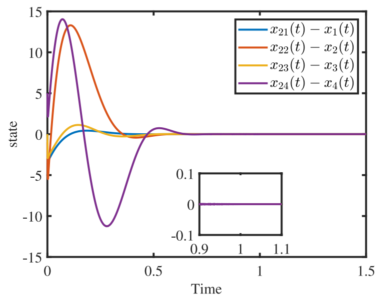

Fig. 4. Trajectory of state error $x_{2,j}-x_j$, $j=1,2,3,4$ on observer 2.

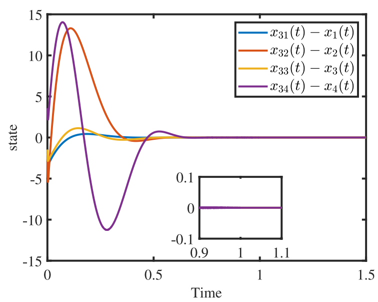

Fig. 5. Trajectory of state error $x_{3,j}-x_j$, $j=1,2,3,4$ on observer 3.

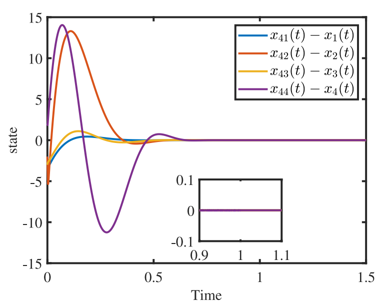

Fig. 6. Trajectory of state error $x_{4,j}-x_j$, $j=1,2,3,4$ on observer 4.

Then, we obtain the simulation results on state error $e_{i,j}=x_{i,j}-x_j$, $i\in\mathcal{N}=\{1,2,3,4,5\}$; $j=1,2,3,4$ for each observer, as shown in Figs. 3–7, from which one can clearly see that the designed distributed prescribed finite-time observer not only guarantees the convergence of the state error on a prescribed time interval $[0,1)$ but also ensures the definite zero-state error on the time interval $[1,1.5)$.

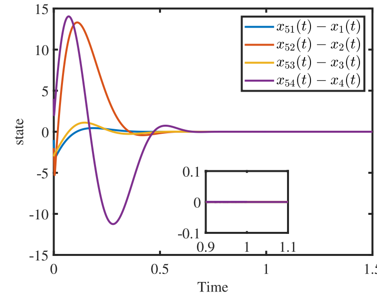

Fig. 7. Trajectory of state error $x_{5,j}-x_j$, $j=1,2,3,4$ on observer 5.

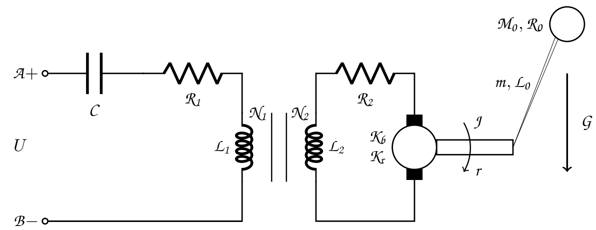

Fig. 8. Schematic of the electromechanical system.

B. Application to Electromechanical System

In this section, we consider an electromechanical system in a mutual inductance transformer circuit, also shown in Fig. 8. The electromechanical model can be seen in [42] and its system dynamics are described by

$$\mathcal{D}\ddot{r}+\mathcal{B}\dot{r}+\mathcal{N}\sin(r)=I$$
$$\mathcal{R}_2 I+\mathcal{L}_2\dot{I}=\mathcal{K}_6\dot{r}$$
$$\frac{1}{C}q+\mathcal{R}_1\dot{q}+\frac{\mathcal{N}_1}{\mathcal{N}_2}\mathcal{L}_2\dot{I}=\mathcal{U}+\Delta\mathcal{U}$$

where $\mathcal{D}=(\mathcal{J}/\mathcal{K}_r)+(m\mathcal{L}_0^2/3\mathcal{K}_r)+([\mathcal{M}_0\mathcal{L}_0^2]/\mathcal{K}_r)+([2\mathcal{M}_0\mathcal{R}_0^2]/5\mathcal{K}_r)$, $\mathcal{N}=(m\mathcal{L}_0\mathcal{G}/2\mathcal{K}_r)+([\mathcal{M}_0\mathcal{L}_0\mathcal{G}]/K_r)$, $\mathcal{B}=(\mathcal{B}_0/\mathcal{K}_r)$, $\mathcal{J}$ is the rotor inertia, m is the link mass, $\mathcal{M}_0$ is the load mass, $\mathcal{L}_0$ is the link length, $\mathcal{R}_0$ is the radius of the load, $\mathcal{G}$ is the gravity coefficient, $\mathcal{B}_0$ is the coefficient of viscous friction at the joint, $r(t)$ is the angular motor position (and thus the position of the load), $I(t)$ is the motor armature current, $q(t)$ is the coulombs in the side of input, $\mathcal{K}_r$ is the coefficient that characterizes the electromechanical conversion of armature current to torque, $\mathcal{R}_1$ and $\mathcal{R}_2$ are the armature resistance, $\mathcal{L}_1$ and $\mathcal{L}_2$ are the armature inductance, $\mathcal{U}$ is the nominal voltage, and $\Delta\mathcal{U}$ represents the discrepancy between the nominal voltage and the actual voltage.

Choosing $x_1=r, x_2=\dot{r}, x_3=(1/\mathcal{D})I, x_4=q-(1/\mathcal{R}_1)\int_0^t e^{-C^{-1}(t-s)}\mathcal{U}(s)ds$, the electromechanical system can be rewritten as (2), where $\delta_1=\delta_2=1, \delta_3=0$ in matrix $A, f_2(x_1,x_2)=-(1/\mathcal{D})\mathcal{B}x_2-(1/\mathcal{D})\mathcal{N}\sin(x_1), f_3(x_1,x_2,x_3)=(\mathcal{K}_6/\mathcal{D}\mathcal{L}_2)x_2-(\mathcal{R}_2/\mathcal{L}_2)x_3, f_4(x_1,x_2,x_3,x_4)=-(1/\mathcal{R}_1C)x_4-(\mathcal{N}_1/\mathcal{R}_1\mathcal{N}_2)\mathcal{L}_2 f_3(x_1,x_2,x_3)$, and $d(t)=(\Delta\mathcal{U}/\mathcal{R}_2)$. In this example, we consider a network of three interacting observers. The measurement output matrices are given as $C_1=(1\ \ 0\ \ 0\ \ 0)$, $C_2=(0\ \ 0\ \ 0\ \ 1)$, and $C_3=0$. The network topology of the three observers is demonstrated in Fig. 9, where the adjacency element $a_{ij}=1$ if $(i,j)\in\mathcal{E}$ and $a_{ij}=0$ otherwise. From Lemma 4, we choose $\varepsilon=([2-\sqrt{3}]/3)$, thus

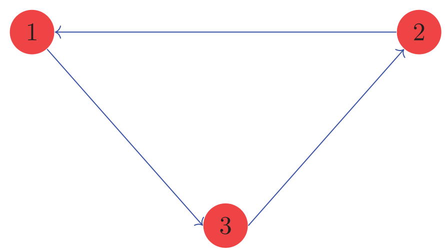

Fig. 9. Network topology of three interacting observers for Example IV-B.

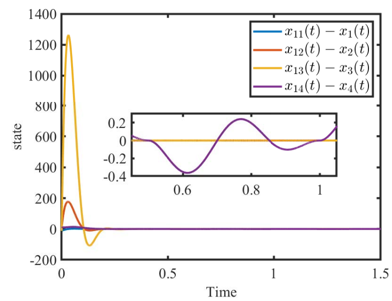

Fig. 10. Trajectory of state error $x_{1,j}-x_j$, $j=1,2,3,4$ on observer 1.

leading to $\gamma = 43$. Employing Lemma 5, matrices $\bar{L}_1, \bar{L}_2$ P_1, and P_2 can be chosen as $\bar{L}_1 = (-3 \;\; -3 \;\; -1 \;\; 0)^T$, $\bar{L}_2 = (0 \;\; 0 \;\; 0 \;\; -1)^T$

$$P_1 = \begin{pmatrix} 18 & 34 & 21 & 0 \\ 34 & 139 & 83 & 0 \\ 21 & 83 & 99 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 22 \end{pmatrix}.$$

Furthermore, let $P_3 = I_4$. Then, following (16) and (17), the observer gain matrices L_1, L_2, K_1, K_2, and K_3 can be determined by choosing $T = 1.5$ s, $T^* = 0.5$ s, and $\bar{k} = 2$. The values of the parameters are chosen as $\mathcal{J} = 1.625 \times 10^3$ kg$\cdot$m^2, $m = 0.506$ kg, $\mathcal{R}_0 = 0.023$ m, $\mathcal{M}_0 = 0.434$ kg, $\mathcal{L}_0 = 0.305$ m, $\mathcal{B}_0 = 16.25 \times 10^3$ N$\cdot$m$\cdot$s/rad, $\mathcal{L}_1 = 15H$, $\mathcal{L}_2 = 10H$ $\mathcal{R}_1 = 5.0\ \Omega$, $\mathcal{R}_2 = 2\ \Omega$, $\mathcal{C} = 5$ F, $\mathcal{K}_r = \mathcal{K}_{\mathcal{B}} = 0.90$ N $\cdot$ m/A, and $\mathcal{N}_1/\mathcal{N}_2 = 2/3$. Moreover, $\mathcal{G}$ is chosen as 10, $\mathcal{U} = 100\sin(6t)$, and $d(t)$ is bounded by 10. Selecting $\mu = 10$, $k = 0, 1, 2$, and we can achieve, for $t \in [0.5k, 0.5(k+1))$

$$L_1 = \begin{pmatrix} -\frac{30}{0.5k-t} \\ -\frac{300}{(0.5k-t)^2} \\ -\frac{1000}{(0.5k-t)^2} \\ 0 \end{pmatrix}, \quad L_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\frac{10}{0.5k-t} \end{pmatrix}, L_3 = 0$$

and

$$K_1 = \begin{pmatrix} -\frac{42.7}{0.5k-t} & 0.02 & 0 & 0 \\ \frac{103.3}{(0.5k-t)^2} & -\frac{8.6}{0.5k-t} & 0.5 & 0 \\ \frac{57.7}{(0.5k-t)^3} & \frac{50.1}{(0.5k-t)^2} & -\frac{8.6}{0.5k-t} & 0 \\ 0 & 0 & 0 & -\frac{430}{0.5k-t} \end{pmatrix}$$

$$K_2 = \begin{pmatrix} -\frac{430}{0.5k-t} & 0 & 0 & 0 \\ 0 & -\frac{430}{0.5k-t} & 0 & 0 \\ 0 & 0 & -\frac{430}{0.5k-t} & 0 \\ 0 & 0 & 0 & -\frac{19.5}{0.5k-t} \end{pmatrix}$$

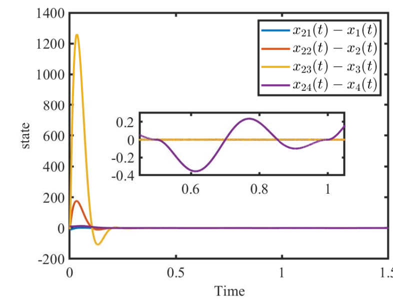

Fig. 11. Trajectory of state error $x_{2,j}-x_j$, $j=1,2,3,4$ on observer 2.

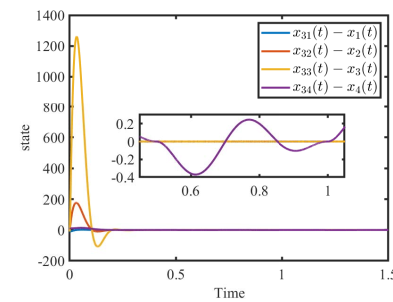

Fig. 12. Trajectory of state error $x_{3,j}-x_j$, $j=1,2,3,4$ on observer 3.

$$K_3 = -\frac{430}{0.5k-t} I_4.$$

Implementing the designed observers, we obtain the simulation results on state error $e_{i,j} = x_{i,} - x_j$, $i = 1, 2, 3; j = 1, 2, 3, 4$ for each observer, as shown in Figs. 10–12. It can be clearly observed that the state errors are bounded by 0.4 on a prescribed time interval [0.5, 1.5).

V. Conclusion

The problem of designing a distributed prescribed finite-time observer for a strict-feedback nonlinear system on a finite-time interval $[0, T)$ over a network of interacting observers has been studied. The prescribed finite-time convergence performance on the state error for each observer has been achieved by employing the time-varying gain $(\mu/[T^* - t])$ on the finite-time interval. To cope with disturbance in the system dynamics, desired observers have also been designed such that the state error between each observer and the system state can reach the prescribed finite-time bounded on the prescribed finite-time interval $[T^*, T)$. The two illustrative examples have been given to verify the effectiveness of the proposed design method.

Appendix A
Proof of Lemma 3

One can observe that for each $p_i \in \{1, 2, \ldots, n\}$, there is exactly one τ_i in $\{1, 2, \ldots, n\}$, which ensures the well-defined mapping. Now, we prove that $\{1, 2, \ldots, n\} \subset \cup_{i=1,\ldots,m}\{p_i, p_i + 1, \ldots, p_i + \tau_i - 1\}$ under the assumed observability of matrix pair (C, A) by resorting to contradiction. Suppose that there exists one number k such that $k \in \{1, 2, \ldots, n\}$,

$\{1, 2, \ldots, k-1\} \subset \cup_{i=1,\ldots,m}\{p_i, p_i+1, \ldots, p_i+\tau_i-1\}$ and $k \notin \cup_{i=1,\ldots,m}\{p_i, p_i+1, \ldots, p_i+\tau_i-1\}$. Apparently, $k \neq p_i, i = 1, 2, \ldots, m$. Denote $\delta_0 = 0$. From the definition of the mapping ϑ, if $\delta_{k-1} = 1$, we can conclude $k \in \{p_i, p_i+1, \ldots, p_i+\tau_i-1\}$ from $k-1 \in \{p_i, p_i+1, \ldots, p_i+\tau_i-1\}$ for some p_i. Thus, $\delta_{k-1} = 0$ and $k \neq p_i, i = 1, 2, \ldots, m$. At this time, all elements in the kth column of $A, A^2, \ldots, A^n$ are equal to 0, and all elements in the kth column of $C_iA, C_iA^2, \ldots, C_iA^n$ are equal to 0. Meanwhile, $k \neq p_i, i = 1, 2, \ldots, m$ which indicates that all elements in the kth column of C are also equal to 0. Thus, the rank of the observability matrix $(C, CA, CA^2, \ldots, CA^n)$ is less than n, which contradicts the observability of (C, A).

Appendix B
Proof of Lemma 4

By virtue of the Cauchy–Schwarz inequality, one has that $(a_1+a_2+\cdots+a_N)^2 = a_1^2 + 2a_1(a_2+\cdots+a_N) + (a_2+\cdots+a_N)^2 \leq a_1^2 + 2a_1(a_2+\cdots+a_N) + (N-1)(a_2^2+\cdots+a_N^2)$. Since for any positive constant η, we have that $2a_1a_i \leq \eta a_1^2 + (1/\eta)a_i^2, \quad i = 2, 3, \ldots, N$, which yields $(a_1+a_2+\cdots+a_N)^2 \leq (2-N+(N-1)\eta-(1/\eta))a_1^2 + N(1-(1/N)(1-[1/\eta]))(a_1^2+a_2^2+\cdots+a_N^2)$. When $\eta \geq 1$, one obtains that $(2-N+(N-1)\eta-(1/\eta))$ is ranging from 0 to $+\infty$. For any positive constant q_1, one can find η such that $(2-N+(N-1)\eta-[1/\eta]) \leq q_1$, and q_2 can be defined as $q_2 = (1/N)(1-[1/\eta])$. Moreover, $q_2 = 0$ only when $\eta = 1$, resulting in $q_1 = 0$.

Appendix C
Proof of Lemma 5

Note that the matrix A can be decomposed into $A = \mathrm{diag}\{A_{i1}, A_{i2}, A_{i3}\}$, where

$$A_{1i} = \begin{pmatrix} 0 & \tilde{A}_{1i} \\ 0 & 0 \end{pmatrix} \; A_{2i} = \begin{pmatrix} 0 & \tilde{A}_{2i} \\ 0 & 0 \end{pmatrix} \; A_{3i} = \begin{pmatrix} 0 & \tilde{A}_{3i} \\ 0 & 0 \end{pmatrix}$$

with $\tilde{A}_{1i} = \mathrm{diag}\{\delta_1, \ldots, \delta_{p_i-1}\}$, $\tilde{A}_{2i} = \mathrm{diag}\{\delta_{p_i}, \ldots, \delta_{p_i+\tau_i-2}\}$, and $\tilde{A}_{3i} = \mathrm{diag}\{\delta_{p_i+\tau_i-1}, \ldots, \delta_{n-1}\}$. If $p_i = 1$ or $p_i+\tau_i = n$, the decomposition of A is less than three parts. However, such a special case can be excluded if one allows that A_{i1} and A_{i3} belong to $\emptyset$. Besides, C_i can be divided into $C_i = (0_{1\times(p_i-1)}, \tilde{C}_i, 0_{1\times(n+1-p_i-\tau_i)})$, where $\tilde{C}_i = (1, 0, \ldots, 0) \in \mathbb{R}^{1\times\tau_i}$.

Recalling that the elements $\delta_{p_i}, \ldots, \delta_{p_i+\tau_i-1}$ in A_{i2} are all equal to 1, for any γ, there exist matrices $\tilde{L}_i$ and $\tilde{P}_i$ and positive constants $\tilde{\alpha}, \tilde{\beta}$ such that [12], [16]

$$\begin{aligned} &\tilde{P}_{\tau_i}\left(A_{2i} + \tilde{L}_{\tau_i}\tilde{C}_i\right) + \left(A_{2i} + \tilde{L}_{\tau_i}\tilde{C}_i\right)^T\tilde{P}_{\tau_i} \leq A_{2i} + A_{2i}^T - \gamma I_{\tau_i} \\ &\tilde{\alpha} I_{\tau_i} \leq \tilde{P}_i H_i + H_i\tilde{P}_i \leq \tilde{\beta} I_{\tau_i} \end{aligned}$$

where $H_i = \mathrm{diag}\{n+1-p_i, n-p_i, \ldots, n+2-p_i-\tau_i\}$. Thus, by choosing $P_i = \mathrm{diag}\{I_{p_i-1}, \tilde{P}_{\tau_i}, I_{n+1-\tau_i-p_i}\}$, $\bar{L}_i = \left(0_{1\times(p_i-1)} \;\; \tilde{L}_{\tau_i}^T \; 0_{1\times(n+1-p_i-\tau_i)}\right)^T$, and properly selecting α, β, inequalities (12) and (13) can be easily verified.

Appendix D
Proof of Theorem 1

For $t \in [0, T^*)$, we introduce a variable

$$e_i = \Phi(X_i - X), \quad i \in \mathcal{N}. \tag{18}$$

From (2), (7), and (14), we have that

$$\begin{aligned} \dot{e}_i = &\frac{\mu}{T^*-t}Ae_i + \frac{\mu}{T^*-t}\bar{L}_iC_ie_i + \frac{1}{T^*-t}He_i \\ &- \frac{\mu}{T^*-t}\gamma P_i^{-1}\sum_{j=1}^{N}a_{ij}(e_i-e_j) + \Delta F_i \end{aligned} \tag{19}$$

where $\Delta F_i = \Phi(F(t, X_i) - F(t, X))$.

Recalling positive matrices $P_i, i \in \mathcal{N}$, the following Lyapunov function candidate can be chosen:

$$V = \sum_{i=1}^{N}\xi_i e_i^T P_i e_i \tag{20}$$

where $\xi_i, i \in \mathcal{N}$ are the positive constants defined in Lemma 1.

The time derivative of V along the trajectory of (19) can be calculated as

$$\begin{aligned} \dot{V}|_{(19)} = &\frac{\mu}{T^*-t}\sum_{i=1}^{N}2\xi_i e_i^T(A+\bar{L}_iC_i)^TP_ie_i \\ &- 2\frac{\mu}{T^*-t}\gamma\sum_{i=1}^{N}\xi_i e^T\sum_{j=1}^{N}a_{ij}(e_i-e_j) \\ &+ \frac{1}{T^*-t}\sum_{i=1}^{N}2\xi_i e_i^TP_iHe_i \\ &+ 2\sum_{i=1}^{N}\xi_i e_i^TP_i\Delta F_i. \end{aligned} \tag{21}$$

Next, each term in the right-hand side of (21) will be handled. Let us start from the first term. It follows from Lemma 5 that for $i = 1, 2, \ldots, m$:

$$\begin{aligned} &2e_i^T(A+\bar{L}_iC_i)^TP_ie_i \\ &\leq 2\|A\|\|e_i\|^2 - \gamma e_i^TM_{\tau_i}e_i \\ &\leq 2\|A\|\|e_i\|^2 - \gamma\|e_{\tau_i}\|^2 \end{aligned} \tag{22}$$

where $e_{\tau_i} = (e_{i,p_i}, e_{i,p_i+1}, \ldots, e_{i,p_i+\tau_i-1})^T$ with $e_{i,j}$ being the jth element of e_i.

Furthermore, it can be seen that for $i = m+1, \ldots, N$

$$2e_i^T(A+\bar{L}_iC_i)^TP_ie_i \leq 2\|A\|\|e_i\|^2. \tag{23}$$

The second term in the right-hand side of (21) can be manipulated as follows:

$$\begin{aligned} &2\sum_{i=1}^{N}\xi_i e^T\sum_{j=1}^{N}a_{ij}(e_i-e_j) \\ &= e^T\tilde{\mathcal{L}} \otimes I_n e \\ &\leq \lambda_2\|e\|^2 - \frac{\lambda_2}{N}\|e_1+e_2+\cdots+e_N\|^2 \end{aligned} \tag{24}$$

where $e = (e_1^T, e_2^T, \ldots, e_N^T)^T$ and $\tilde{\mathcal{L}}$ is defined in Lemma 1.

It follows from Lemma 5, that the third term in the right-hand side of (21) satisfies:

$$\sum_{i=1}^{N} \xi_i e_i^T (P_i H + H P_i) e_i \geq \sum_{i=1}^{N} \xi_i \alpha \|e_i\|^2. \tag{25}$$

Before estimating the fourth term $2\sum_{i=1}^{N} \xi_i e_i^T P_i \Delta F_i$, we discuss the bound of ΔF_i. From Assumption 1, when $\mu \geq T^*$, we have $(\mu^{n+1-j}/[(T^*-t)^{n+1-j}])|f_j(x_1,\ldots,x_j) - f_j(x_{i,1},\ldots,x_{i,j})| \leq \theta((\mu^n/[(T^*-t)^n])|x_1 - x_{i,1}| + \cdots + (\mu^{n+1-j}/[(T^*-t)^{n+1-j}])|x_j - x_{i,j}|) \leq \theta(|e_{i,1}| + \cdots + |e_{i,j}|) \leq \theta\sqrt{n}\|e_i\|$ for $i \in \mathcal{N}$, and $j = 1, 2, \ldots, n$, which leads to

$$2e_i^T P_i \Delta F_i \leq 2n\theta \|P_i\| \|e_i\|^2. \tag{26}$$

Combining (22)–(26) with (21), we have that

$$\begin{aligned}\dot{V}|_{(19)} \leq &-\frac{\mu}{T^*-t}(\gamma\lambda_2 - 2\|A\|)\|e\|^2 - \frac{\mu}{T^*-t}\gamma\bar{\xi}\sum_{k=1}^{m}\|e_{\tau_k}\|^2 \\ &+ \frac{\mu}{T^*-t}\gamma\frac{\lambda_2}{N}\|e_1 + e_2 + \cdots + e_N\|^2 \\ &+ \frac{1}{T^*-t}\alpha N\|e\|^2 + p\theta\|e\|^2\end{aligned}$$

where $p = 2n\max\{\|P_1\|,\ldots,\|P_N\|\}$, $\bar{\xi} = \min\{\xi_1,\ldots,\xi_N\}$, and $\xi_i \leq N$ is employed.

Due to the fact that the matrix pair (C, A) is observable, it follows from Lemma 3 that: $\{1, 2, \ldots, n\} \subset \cup_{i=1,\ldots,m}\{p_i, p_i + 1, \ldots, p_i + \tau_i - 1\}$, which indicates that for any $j \in \{1, 2, \ldots, n\}$, there exists at least one k_j such that $(k_j, j) \in \cup_{i=1,\ldots,m}\{(i, p_i), (i, p_i+1), \ldots, (i, p_i + \tau_i - 1)\}$. Thus, we have $\sum_{i=1}^{m}\|e_{\tau_i}\|^2 \geq \sum_{j=1}^{n}|e_{k_j,j}|^2$.

From Lemma 4, by choosing $q_1 = (N/\lambda_2)\bar{\xi}$, there exists a positive constant $q_2 = (\varepsilon/\lambda_2)$ such that for $j = 1, 2, \ldots, n$, $(e_{1,j} + e_{2,j} + \cdots + e_{N,j})^2 \leq (N/\lambda_2)\bar{\xi}|e_{k_j,j}|^2 + N(1 - [\varepsilon/\lambda_2])(|e_{1,j}|^2 + \cdots + |e_{N,j}|^2)$, and $(\lambda_2/N)\|e_1 + e_2 + \cdots + e_N\|^2 \leq (\lambda_2 - \varepsilon)\|e\|^2 + \bar{\xi}\sum_{j=1}^{n}|e_{k_j,j}|^2$.

Therefore, one can obtain

$$\begin{aligned}\dot{V}|_{(19)} \leq &-\frac{\mu}{T^*-t}\gamma\varepsilon\|e\|^2 + 2\frac{\mu}{T^*-t}\|A\|\|e\|^2 \\ &+ \frac{1}{T^*-t}\alpha N\|e\|^2 + p\theta\|e\|^2.\end{aligned}$$

Choosing $\gamma \geq ([2\|A\| + 1]/\varepsilon)$ and $\mu \geq \alpha N + 1$, one can find a positive constant ζ such that

$$\dot{V}|_{(19)} \leq -\frac{1}{T^*-t}\rho V + \zeta\theta V \tag{27}$$

where $\rho^{-1} = \max\{\xi_1\lambda_{\max}(P_1), \ldots, \xi_N\lambda_{\max}(P_N)\}$ with $\lambda_{\max}(P)$ denoting the maximal eigenvalues of P.

Thus, one obtains that

$$V(t) \leq V(0)\exp\left(\int_0^t\left(-\frac{\rho}{T^*-s} + \zeta\theta\right)ds\right) \tag{28}$$

yielding $\lim_{t\to T^*} V(t) = 0$.

From the definition of V, there is a positive constant ν such that $|x_{j,i} - x_i| = ([(T^*-t)^{n+1-i}]/\mu^{n+1-i})|e_{j,i}| \leq \|e\| \leq \nu\sqrt{V}$, holds for any $i = 1, 2, \ldots, n$ and $j \in \mathcal{N}$, which leads to

$$\lim_{t\to T^*}\|X_j - X\| = 0, \quad j \in \mathcal{N}.$$

Since X and $X_j, j \in \mathcal{N}$ are continuous in the interval $[0, T)$, one can obtain

$$\|X_j(T^*) - X(T^*)\| = \lim_{t\to T^*}\|X_j - X\| = 0. \tag{29}$$

Next, we analyze the following error dynamics in terms of $\eta_i = X_i - X, i \in \mathcal{N}$ on $[T^*, T)$:

$$\dot{\eta}_i = A\eta_i + \delta F_i, \qquad t \in [T^*, T) \tag{30}$$

where $\delta F_i = F(t, X_i) - F(t, X)$.

Note that $F(\cdot,\cdot)$ satisfies Assumption 1, and η_i is the equilibrium point. One can achieve that

$$\eta_i(t) \equiv 0, \quad t \in [T^*, T) \tag{31}$$

for $i \in \mathcal{N}$ from $\eta_i(T^*) = 0$ in (29).

Appendix E
Proof of Theorem 2

Consider the state transformation $e_i = \Phi(X_i - X), \quad i \in \mathcal{N}$. Then, it follows from (16) and (17) that:

$$\begin{aligned}\dot{e}_i = &\frac{\mu}{kT^*-t}Ae_i + \frac{\mu}{kT^*-t}\bar{L}_iC_ie_i + \frac{1}{kT^*-t}He_i \\ &- \frac{\mu}{kT^*-t}\gamma P_i^{-1}\sum_{j=1}^{N}a_{ij}(e_i - e_j) + \Delta F_i + \frac{\mu}{kT^*-t}Bd(t)\end{aligned} \tag{32}$$

where $\Delta F_i = \Phi(F(t, X_i) - F(t, X))$.

Choose the Lyapunov function candidate as

$$V = \sum_{i=1}^{N}\xi_i e_i^T P_i e_i \tag{33}$$

where $\xi_i, i \in \mathcal{N}$ are the positive constants defined in Lemma 1. When $\mu \geq \max\{\alpha N + 1, T, \rho^{-1}\}$ and $\gamma \geq ([2\|A\| + 1 + \rho^{-1}]/\varepsilon)$, the time derivative of V along the trajectory of (32) can be computed as

$$\dot{V}|_{(32)} \leq -\frac{\mu}{kT^*-t}\rho V + \zeta\theta V + \iota\frac{\mu}{kT^*-t}|d(t)|\sqrt{V} \tag{34}$$

where ι is a positive constant, and ρ, ζ are defined in (27).

Denoting $\omega(t) = (kT^*-t)^{-1}e^{-(1/2)\zeta\theta(t-(k-1)T^*)}\sqrt{V}$, we can obtain $\dot{\omega} \leq (1/2)\tilde{d}(kT^*-t)^{-2}$, where $\tilde{d} = \iota\bar{d}\mu$ with $\bar{d}$ being the bounded of $d(t)$. Thus, one has $\omega(t) - \omega((k-1)T^*) \leq \tilde{d}((kT^*-t)^{-1} - (T^*)^{-1})$, which follows from $\omega((k-1)T^*) = (T^*)^{-1}\sqrt{V((k-1)T^*)}$ that:

$$\sqrt{V(t)} \leq e^{\frac{1}{2}\zeta\theta T^*}\sqrt{V((k-1)T^*)} + \tilde{d}e^{\frac{1}{2}\zeta\theta T^*}.$$

Note that there exist positive constants h_1 and h_2 such that

$$h_1\frac{1}{kT^*-t}\|\eta\| \leq \sqrt{V(t)} \leq h_2\frac{1}{(kT^*-t)^n}\|\eta\|.$$

As a result, one obtains that

$$\begin{aligned}\|\eta(t)\| \leq &e^{\frac{1}{2}\zeta\theta T^*}h_2\frac{1}{h_1(T^*)^n}\left(kT^*-t\right)\|\eta\left((k-1)T^*\right)\| \\ &+ \frac{1}{h_1}\left(kT^*-t\right)\tilde{d}e^{\frac{1}{2}\zeta\theta T^*}\end{aligned}$$

yielding $\|\eta(kT^*)\| = 0$, $k = 1, 2, \ldots, \bar{k}$. Therefore, when $t \in [T^*, T)$, we have that $\|\eta(t)\| \leq (1/h_1)(kT^*-t)\tilde{d}e^{(1/2)\zeta\theta T^*}$ which leads to (15) by choosing $\Omega = (1/h_1)\tilde{d}e^{(1/2)\zeta\theta T}$.

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Le Chang received the B.Sc. degree in mathematics from Shandong University, Jinan, China, in 2012. He is currently pursuing the Ph.D. degree with the School of Control Science and Engineering.

From 2017 to 2019, he was a visiting Ph.D. student with the Swinburne University of Technology, Melbourne, VIC, Australia. His current research interests include distributed estimation and control of nonlinear systems.

Qing-Long Han (M'09–SM'13–F'19) received the B.Sc. degree in mathematics from Shandong Normal University, Jinan, China, in 1983, and the M.Sc. and Ph.D. degrees in control engineering and electrical engineering from the East China University of Science and Technology, Shanghai, China, in 1992 and 1997, respectively.

From 1997 to 1998, he was a Postdoctoral Researcher Fellow with the Laboratoire d'Automatique et d'Informatique Industielle (currently, Laboratoire d'Informatique et d'Automatique pour les Systémes), École Supérieure d'Ing'enieurs de Poitiers (currently, École Nationale Supérieure d'Ingénieurs de Poitiers), Université de Poitiers, Poitiers, France. From 1999 to 2001, he was a Research Assistant Professor with the Department of Mechanical and Industrial Engineering, Southern Illinois University Edwardsville, Edwardsville, IL, USA. From 2001 to 2014, he was a Laureate Professor, an Associate Dean (Research and Innovation) with the Higher Education Division, and the Founding Director of the Centre for Intelligent and Networked Systems at Central Queensland University, Rockhampton, QLD, Australia. From 2014 to 2016, he was a Deputy Dean (Research), with the Griffith Sciences, and a Professor with the Griffith School of Engineering, Griffith University, Gold Coast, QLD, Australia. In 2016, he joined the Swinburne University of Technology, Melbourne, VIC, Australia, where he is currently a Pro Vice-Chancellor (Research Quality) and a Distinguished Professor. In 2010, he was appointed as a Chang Jiang (Yangtze River) Scholar Chair Professor by the Ministry of Education, China. His current research interests include networked control systems, multiagent systems, time-delay systems, complex dynamical systems, and neural networks.

Prof. Han was a recipient of the Highly Cited Researcher Award by Clarivate Analytics (formerly, Thomson Reuters). He is an Associate Editor of several international journals, including the IEEE TRANSACTIONS ON CYBERNETICS, the IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, the IEEE TRANSACTIONS ON INDUSTRIAL INFORMATICS, the *IEEE Industrial Electronics Magazine*, the *IEEE/CAA Journal of Automatica Sinica*, *Control Engineering Practice*, and *Information Sciences*. He is a fellow of the Institution of Engineers Australia.

Xiaohua Ge (M'18) received the B.Eng. degree in electronics and information engineering from Nanchang Hangkong University, Nanchang, China, in 2008, the M.Eng. degree in control theory and control engineering from Hangzhou Dianzi University, Hangzhou, China, in 2011, and the Ph.D. degree in computer engineering from Central Queensland University, Rockhampton, QLD, Australia, in 2014.

From 2011 to 2013, he was a Research Assistant with the Centre for Intelligent and Networked Systems, Central Queensland University, where he was a Research Fellow in 2014. From 2015 to 2017, he was a Research Fellow with the Griffith School of Engineering, Griffith University, Gold Coast, QLD, Australia. He is currently a Lecturer with the School of Software and Electrical Engineering, Swinburne University of Technology, Melbourne, VIC, Australia. His current research interests include distributed estimation over sensor networks, distributed coordination in multiagent systems, and security and privacy preserving in cyber-physical systems.

Chenghui Zhang (M'14–SM'18) was born in Shandong, China, in 1963. He received the B.Sc. and M.Sc. degrees in automation engineering from the Shandong University of Technology, Jinan, China, in 1985 and 1988, respectively, and the Ph.D. degree in control theory and operational research from Shandong University, Jinan, China, in 2001.

In 1988, he joined Shandong University, where he is currently a Full Professor with the School of Control Science and Engineering, and the Director of Research Center of Power Electronics Energy Saving Technology and Equipment of the Chinese Education Ministry. He was selected as a Changjiang Scholar of the Education Ministry in 2009, and the Taishan Scholar of Shandong Province in 2009, respectively. His current research interests include optimal control of engineering, power electronics, and energy-saving techniques.

Xianfu Zhang (M'17) received the M.Sc. degree in fundamental mathematics from the School of Mathematics Sciences, Shandong Normal University, Jinan, China, in 1999, and the Ph.D. degree in operational research and control from the School of Mathematics, Shandong University, Jinan, in 2005.

From 1999 to 2011, he was with the School of Science, Shandong Jianzhu University, Jinan. He joined the School of Control Science and Engineering, Shandong University in 2012, where he is currently a Professor. From 2008 to 2009, he was a Visiting Scholar with the Department of Mechanical Engineering, Ecole Polytechnique de Montreal, Montreal, QC, Canada. From 2009 to 2010 and in 2012, he was a Research Assistant with the City University of Hong Kong, Hong Kong. He has been the Taishan Scholar of Shandong Province of China since 2019. His current research interests include nonlinear systems, multiagent systems, and time-delay systems.